Metameric Observers: A Monte Carlo Approach
S-Cone Fundamentals
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Variability of Spectral Tristimulus Values

Isadore Nimeroff, Joan R. Rosenblatt, and Mary C. Dannemiller

(July 11, 1961)

As the spectral tristimulus values of the CIE Standard Observer System for Colorimetry are measurable quantities, their variabilities should be known. This paper describes a procedure for deriving "within" and "between" variances and covariances in the spectral tristimulus values, based on color-matching data for individual observers. The "within" variances are based on the replications of color-mixture data by an observer. The "between" variances are based on differences among the color-mixture data of individual observers. A statistical model is given for the system in which the experimental data are obtained. Formulas for expected values (means), variances, and covariances are developed. Variances and covariances belonging to different sources of uncertainties in the experimental data are considered. A procedure is developed for determining the uncertainties in the constants of a linear transformation to a system analogous to the present CIE system. The formulas for variances and covariances after linear transformation are given, for a rigorous empirically-based choice, and also for an arbitrary choice of transformation constants. The complete standard observer system for every 10 mμ consisting of means, variances, and covariances derived from an arbitrary transformation, is listed. The between-observer variabilities are found to be about 10 percent of the averages of the color-mixture data and the average ratio of the between-observer variabilities to the within-observer variabilities is found to be about 5.7.

1. Introduction

Since 1931 the International Commission on Illumination has recommended the use of a Standard Observer System for Colorimetry [1]. This system defines the manner in which spectral data for materials are to be reduced to three numbers, called tristimulus values, that describe colors of emitted, reflected, or transmitted lights. The defining equations for these tristimulus values are:

\[
\begin{align*}
T_x &= \int x(\lambda) d\lambda, \\
T_y &= \int y(\lambda) d\lambda, \\
T_z &= \int z(\lambda) d\lambda,
\end{align*}
\]

where \( x(\lambda), y(\lambda), \) and \( z(\lambda) \) are the spectral luminous efficiency functions for the standard observer. For perfect colors, the spectral quantities, they are subject to measurement uncertainty. Nimeroff [2,3] has treated, by means of propagation of error theory, the manner in which variabilities in \( T_x, T_y, \) and \( T_z \) affect the chromaticity coordinates, \( x, y, \) and \( z. \)

The general problem and several special cases of propagation of errors in tristimulus colorimetry have been treated by Nimeroff [3]. In that treatment the mean spectral tristimulus values, \( \bar{\bar{x}}, \bar{\bar{y}}, \) and \( \bar{\bar{z}} \), were estimated by averaging the mean CIE (17 standard observer) color-mixing data. 

Stiles [2] and 100 field pilot
Modeling Observer Metamerism through Monte Carlo Simulation

Abstract:
Metameric color matches depend on the observer's color matching functions. Data were collected on observer variability in typical metameric matches. A Monte Carlo simulation, using a model of color matching functions and physiological data, was performed to derive a complete colorimetric system capable of predicting inter-observer variability in addition to mean color matches.

Monte Carlo Model:
For a CIE 1931 Standard Colorimetric Observer

\[ x'(\lambda) = 10^{-k_1 x(\lambda)} \left[ k_{33} L(\lambda) + k_{34} M(\lambda) + k_{35} S(\lambda) \right] \]

\[ y'(\lambda) = 10^{-k_1 y(\lambda)} \left[ k_{33} L(\lambda) + k_{34} M(\lambda) + k_{35} S(\lambda) \right] \]

\[ z'(\lambda) = 10^{-k_1 z(\lambda)} \left[ k_{33} L(\lambda) + k_{34} M(\lambda) + k_{35} S(\lambda) \right] \]

Typical Results:

Monte Carlo Results:

Monte Carlo Experiment:
- 10,000 Sets of Color Matching Functions Generated
- Mean and Covariance Functions Established
- Standard Error Propagation to CIELAB Covariance Matrices for Observed Metamers
- Predicted Covariance Dependent upon Metameric Properties

Acknowledgements / References:

Conclusions:
- Observer Variability in Practical Color Matching is Significant
- Previously Published Techniques Underpredict Variability
- A Monte Carlo Model Produced Better Results
- Further Data and Model Refinement are Required
CIE 2006

\[ \bar{l}(\lambda) = \alpha_{i,l}(\lambda) \cdot 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)} \]

\[ \bar{m}(\lambda) = \alpha_{i,m}(\lambda) \cdot 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)} \]

\[ \bar{s}(\lambda) = \alpha_{i,s}(\lambda) \cdot 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)} \]
CIE 2006

Cone Absorptivity Spectra

\[
\bar{l}(\lambda) = \alpha_{i,l}(\lambda) \cdot 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)}
\]

\[
\bar{m}(\lambda) = \alpha_{i,m}(\lambda) \cdot 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)}
\]

\[
\bar{s}(\lambda) = \alpha_{i,s}(\lambda) \cdot 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)}
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CIE 2006

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CIE 2006

Cone Absorptivity Spectra

\[ l(\lambda) = \alpha_{i,l}(\lambda) \cdot 10^{-D_{\tau,\max,macula} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,ocul}(\lambda)} \]

\[ m(\lambda) = \alpha_{i,m}(\lambda) \cdot 10^{-D_{\tau,\max,macula} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,ocul}(\lambda)} \]

\[ s(\lambda) = \alpha_{i,s}(\lambda) \cdot 10^{-D_{\tau,\max,macula} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,ocul}(\lambda)} \]

Ocular Media Density

\[ f(\text{age}) \]

Macular Density

\[ f(\text{field size}) \]
CIE 2006

Cone Absorptivity Spectra

\[ l(\lambda) = \alpha_{i,l}(\lambda) \cdot f(\text{field size}) \]

\[ m(\lambda) = \alpha_{i,m}(\lambda) \]

\[ s(\lambda) = \alpha_{i,s}(\lambda) \]

Ocular Media Density f(age)

Normalized absorbance

Wavelength (nm)

Violet  Blue  Cyan  Green  Yellow  Red

420  498  534  564

S  R  M  L
CIE 2006 Model

\[ l(\lambda) = \alpha_i l(\lambda) \times 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)} \]

\[ m(\lambda) = \alpha_i m(\lambda) \times 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)} \]

\[ s(\lambda) = \alpha_i s(\lambda) \times 10^{-D_{\tau,\text{max,macula}} \cdot D_{\text{macula,relative}}(\lambda) - D_{\tau,\text{ocul}}(\lambda)} \]

Ocular Media Density \( f(\text{age}) \)

Macular Density \( f(\text{field size}) \)
Monte Carlo ...
Monte Carlo ...

\[ f(x) = \frac{b^2}{\theta^6} (\frac{x-\frac{1}{b}}{\theta})^{(b-1)} \cdot e^{-\left(\frac{x-\frac{1}{b}}{\theta}\right)^6} \]

Weibull Distributions
Randomly Select

- Lens
- Macula
- L, M, & S Cones
- Build Cone Fundamentals
- Compute Other CMFs
Randomly Select

- Lens
- Macula
- L, M, & S Cones

Build Cone Fundamentals

Compute Other CMFs

But How???
Randomly Select

- Lens
- Macula
- L, M, & S Cones
- Build Cone Fundamentals
- Compute Other CMFs

But How???

Select each component magnitude from appropriate random distributions....
Pokornoy & Smith Lens Model

- Two Components; One Age Dependent
- Two Functions: $20 \leq A \leq 60$, $A > 60$
- $A$ Selected from Census Data
- Randomly Perturbed to Model Variance within Age

$$T_l = T_{L1}[1 + 0.02(A - 32)] + T_{L2}$$

$$T_l = T_{L1}[1.56 + 0.0667(A - 60)] + T_{L2}$$
Lens (20, 40, 60 yo)
Macular Density Model

- Baseline of Bone et al.
- St. Dev. Estimate of Berendschot and van Norren
- Monte Carlo Scaling Model
- $\delta$ from a Normal Distribution

\[
D_m(\lambda) = D_{m,Bone}(\lambda) \left[ \frac{0.352 + \delta}{0.352} \right]
\]
Macula ± 3 sigma
Cones
Cones

- Genotypes Suggest Discrete Responsivities
Cones

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- Other Variables: Pigment Density, Cone Morphology
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- Phenotypes Show Continuous Variation (probabilities unknown)
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Cones

- Genotypes Suggest Discrete Responsivities
- Other Variables: Pigment Density, Cone Morphology
- Phenotypes Show Continuous Variation \( (probabilities \ unknown) \)
- Stockman and Sharpe Functions Selected for Baseline \( (similar\ to\ CIE\ 2006) \)
Cone Variation

- Random Offsets in Wavenumber
  - L: 51 cm\(^{-1}\) (~1.6 nm)
  - M: 89 cm\(^{-1}\) (~2.5 nm)
  - S: NA for S
Cones ± 3 sigma
Components ...
1000 Observers
Stiles-Burch Normalized

Fig. 3(5.5.6). 10° color-matching functions of 49 observers participating in the Stiles-Burch (1959) experiment. All functions refer to primary stimuli at $m_B = 15,500$, $m_G = 19,000$, and $m_B = 22,500$ cm$^{-1}$. 
Effects on S Cones

- Stockman & Sharpe (1999)
Color Checker Metamers
Comparing Metamers

Left Pair: CIE 2° Observer *(they match, by definition)*

Right Pair: 95th Percentile Observer by Color Difference *(they often don’t)*
Comparing Metamers

Differences large enough, they could be name changes.

Std. Obs. sees 2 pinks, I see an orange and a pink.

Left Pair: CIE 2° Observer *(they match, by definition)*
Right Pair: 95th Percentile Observer by Color Difference *(they often don’t)*
Simulated Appearance Distributions
Data

- Matches Well With ...
  - Alfvin and Fairchild (1995)
  - Sarkar et al. Categories
  - Asano et al. Preliminary Results
  - Informal Observations / Demonstrations
FIG. 5. Intra-observer, and inter-observer, cyan-transparency color matches relative to the 1931 CIE Standard Colorimetric Observer matchpoint located at the origin of a CIE plane. The CIE Standard Colorimetric Observer color-matching functions accurately predicted six of the fourteen inter-observer mean color-matches; and the CIE Supplemental Standard Colorimetric Observer color-matching functions accurately predicted only one of the fourteen inter-observer mean color-matches. The Stiles–Burch and CIE Standard Colorimetric Observer color-matching functions outperformed the CIE Supplemental Standard Colorimetric Observer color-matching functions in terms of predicting the mean color-match for a population of color normal observers. The relatively poor performance of the CIE Supplemental Standard Colorimetric Observer color-matching function is not unexpected, considering the fact that the experimental color-matching stimulus was restricted to a visual field. Although the standard observer color-matching functions were not able to predict the mean color-matches of the group of twenty observers for every sample, it is noteworthy that in every case the predicted color-matches were contained in the ellipsoids that defined the 95% confidence regions at the level for the sample distributions of inter-observer color-matches, as shown in Table V. Therefore, it can be said that the hypothetical color-matches determined with the standard observers are representative of a member of the population of inter-observer color-matches determined in this experiment.

Table VI shows the results of Hotelling's T2 test for comparing the mean color-matches bound by: (a) both 95% bivariate confidence ellipses of the sample distribution (outer ellipse), and the single observer with the hypothetical mean color-matches sample mean (inner ellipse). (b) 95% bivariate confidence ellipses of the sample distribution, and the sample mean; The checkmarks ("\) indicate that the mean color-matches are not significantly different from the specified standard distribution, and the sample mean. The 1931 CIE Standard Colorimetric Observer at the level. The means were found to...
Categories ...

1 Macula
2 Lenses (age)
  1 S
  2 M
  2 L

8 Categories!
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1. Introduction

Since 1931 the International Commission on Illumination has recommended the use of a Standard Observer System for Colorimetry [1]. This system defines the manner in which spectral data for materials are to be reduced to three numbers, called tristimulus values, that describe colors of emitted, reflected, or transmitted lights. The defining equations for these tristimulus values are:

\[ X = \int \frac{\partial E}{\partial \lambda} N_\lambda T_\lambda d\lambda = \sum_{\lambda} \bar{x}_\lambda N_\lambda T_\lambda d\lambda \]
\[ Y = \int \frac{\partial E}{\partial \lambda} N_\lambda T_\lambda d\lambda = \sum_{\lambda} \bar{y}_\lambda N_\lambda T_\lambda d\lambda \]
\[ Z = \int \frac{\partial E}{\partial \lambda} N_\lambda T_\lambda d\lambda = \sum_{\lambda} \bar{z}_\lambda N_\lambda T_\lambda d\lambda \]

The quantities \( \bar{x}_\lambda \), \( \bar{y}_\lambda \), and \( \bar{z}_\lambda \) are called spectral tristimulus values and are intended to be descriptive of the spectral-light response of the average human observer with normal color vision. The quantity \( N_\lambda \) describes the spectral emittance of light sources and the quantity \( T_\lambda \) describes the spectral character of the reflecting or transmitting materials.

Tristimulus values are usually reduced to chromaticity coordinates by the equations:

\[ x = X/S, \quad y = Y/S, \quad \text{and} \quad z = Z/S, \]

where \( S \) is the sum of the tristimulus values \( X, Y, \) and \( Z \). As \( \bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda, N_\lambda, \) and \( T_\lambda \) are measured quantities, they are subject to measurement uncertainty. Nimeroff [2,3] has treated, by means of propagation of error theory, the manner in which variabilities in \( T_\lambda \) and in \( N_\lambda \) affect the chromaticity coordinates, \( x, y, \) and \( z \).

The general problem and several special cases of propagation of errors in tristimulus colorimetry have been treated by Nimeroff [3]. In that treatment the mean spectral tristimulus values, \( \bar{x}, \bar{y}, \) and \( \bar{z} \), were estimated by averaging the mean CIE (17 observers) and mean Stiles' 2\( ^o \) and 10\( ^o \)-field pilot data (10 observers each). The variances in these values were estimated in the usual manner by using deviations of these three mean data from the estimated overall mean values; the covariances were ignored. The variances as well as the covariances should, however, be more fundamentally estimated; that is, they should be estimated from differences among color-mixture functions of individual observers. Such data became available in 1959. This paper describes how this fundamental estimation of the between-observer variances and covariances may be made for the 10\( ^o \)-field color-mixture data of the 53 observers of Stiles-Burch [4] and for the 27 observers of Speranskaya [5], and gives estimates of the average within-observer variances and covariances of two observers, one with 4 and the other with 5 replications. The estimates of covariances are developed on the basis of the data of the 53 observers of Stiles-Burch.

2. Statistical Model

Fundamental color-matching data are obtained on a device where an observer is presented two fields which he is asked to color-match, by adjusting the
Future ...

- Nimeroff et al. (1961)

- "A complete standard observer system should contain not only the mean spectral tristimulus functions ... but also the variances and covariances of these functions ..."

- Coming Soon & Looks Very Promising
It’s not what you look at that matters, it’s what you see

-Henry David Thoreau