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One Set of Munsell Re-renotations

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Munsell Color Company, Inc.  
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Baltimore, Maryland 21218

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U.S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS



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Introduction

Since 1955 the Munsell Color Company has participated in the work of the Committee on Uniform Color Scales, Optical Society of America, under the chairmanship of Dr. Deane B. Judd of this Bureau. During this time the committee has completed four basic preliminary experiments summarized in Table 1 taken from the Progress Report for O.S.A. Committee on Uniform Color Scales.<sup>1/</sup>

The fourth basic experiment (Chromaticness Spacing for Munsell Value 6) was supplemented by two auxiliary experiments: one to determine how many Munsell chroma steps at value 6/ it takes to have the same perceptual importance as one value step for neutral grays near N 6/, the other to determine the Munsell value of the gray appearing to have the same lightness as a sampling of the 43 chips of nominal value 6/. The first auxiliary experiment indicated that 4 chroma steps are perceptually equivalent to one value step under the observing conditions used.<sup>1/</sup> The results of the second auxiliary experiment have been described in a paper by Wyszecki,<sup>2/</sup> entitled Correlate for Lightness in Terms of CIE Chromaticity Coordinates and Luminous Reflectance.

The experimental results of the fourth basic experiment were checked against four theoretical models summarized in Figs. 1 through 4. The members of the committee most interested in these models carried out the checks: CIE  $U^*V^*W^*$  space checked by Wyszecki, Glasser-Reilly Cube-Root Space checked by Reilly, Munsell Renotation Space checked by Nickerson, and MacAdam-Friele (1965) Space checked by MacAdam.

The luminous reflectances (Y) and chromaticity coordinates (x,y) of the 43 test samples were computed for CIE source C from four independent spectrophotometric determinations and the average values are given in Table 2 together with the corresponding Munsell renotations.

C I E U\* V\* W\* Space

$$W^* = 25Y^{1/3} - F7, \quad 1 < Y < 100$$

$$U^* = 13 W^*(u - u_0)$$

$$V^* = 13 W^*(v - v_0)$$

$$u = 4X / (X + 15Y + 3Z)$$

$$v = 6Y / (X + 15Y + 3Z)$$

$$\Delta E = \left[ \overline{\Delta U^*}^2 + \overline{\Delta V^*}^2 + \overline{\Delta W^*}^2 \right]^{1/2}$$

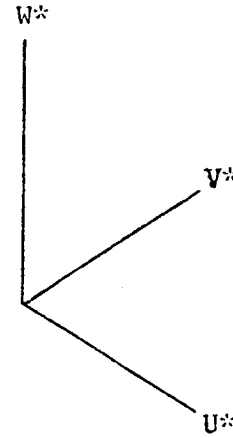


Fig. 1.

Glasser-Reilly Cube-Root Space

$$L = 25.29G^{1/3} - 18.38$$

$$a = 106.0 (R^{1/3} - G^{1/3})$$

$$b = 42.34 (G^{1/3} - B^{1/3})$$

$$R = 1.1085X + 0.0852Y - 0.1454Z$$

$$G = -0.0010X + 1.0005Y + 0.0004Z$$

$$B = -0.0062X + 0.0394Y + 0.8192Z$$

$$\Delta E = \left( \overline{\Delta a}^2 + \overline{\Delta b}^2 + \overline{\Delta L}^2 \right)^{1/2}$$

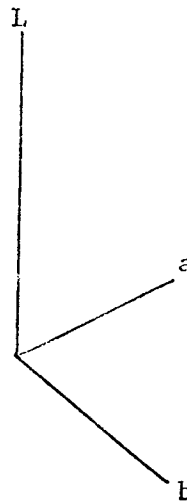


Fig. 2.

Munsell Renotation Space

$$\Delta E = [2C_1 C_2 (1 - \cos 3.6 \Delta H) + \frac{\Delta C^2}{3} + \frac{6\Delta V^2}{4}]^{1/2}$$

(Godlove)

$$= (C/5)(2\Delta H) + 3\Delta C + 6\Delta V$$

(Nickerson Index of Fading)

$$dE = \{ [(C/5)(1.75 \Delta H)]^2 + \frac{\Delta C^2}{3} + \frac{6\Delta V^2}{4} \}^{1/2}$$

(Committee 1966)

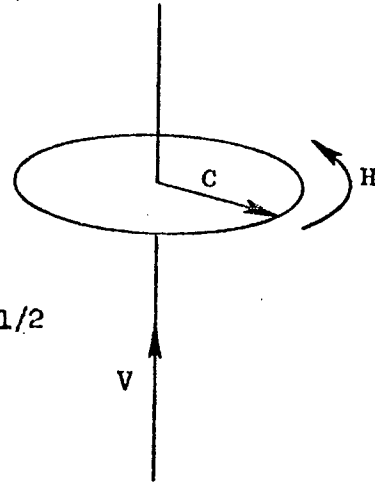


Fig. 3.

MacAdam-Friele (1965)

$$\Delta E = (C_{11}\Delta R^2 + 2C_{12}\Delta R \Delta G + C_{22} \Delta G^2 + 2C_{23}\Delta G \Delta B + C_{33}\Delta B^2 + 2C_{13}\Delta R \Delta G)^{1/2}$$

$$R = 0.4822X + 0.1388Y - 0.0748Z$$

$$G = -0.3869X + 1.2908Y + 0.0748Z$$

$$B = \quad \quad \quad 0.2859Z$$

$$C_{ij} = f(R, G, B; D, F, E, H, J, f)$$

where D, F, E, H, J, f are constants to be optimized for each set of experimental data.

Fig. 4.

Perceived V Minus Measured V  
is indicated below the line  
within each "sample".

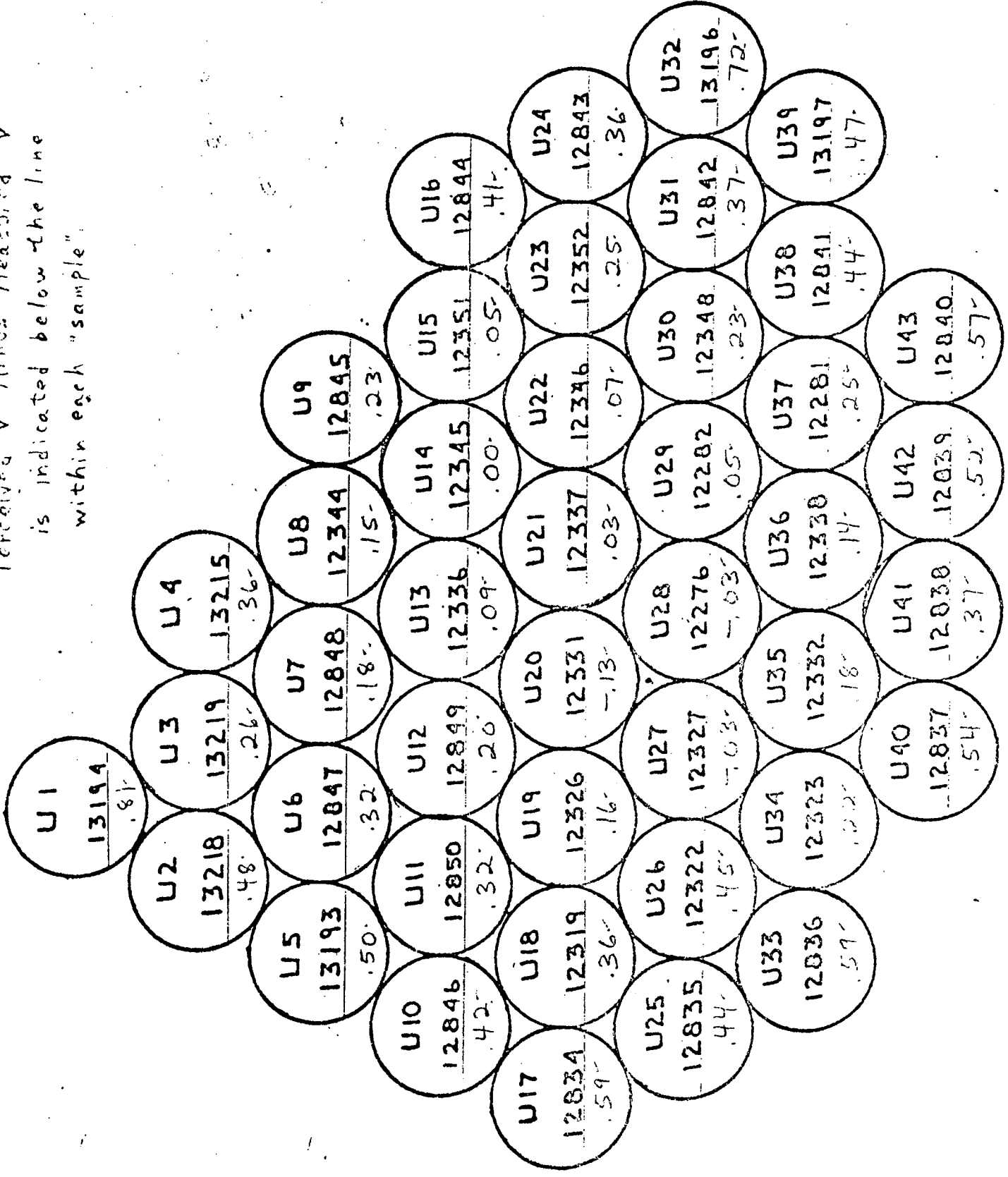


FIG. 5.

The 43 test samples were chosen with the intention of making as uniform a sampling (triangular) as possible of the value 6/ plane. The 107 differences, observed by 70 observers judging simply which of two differences presented is the larger, are identified by sample number in Table 3. Of these 107 differences 104 are between nearest neighbors; three, called double differences and marked with an asterisk, are between samples having one sample between them in the triangular array. Fig. 5 shows schematically this triangular array, the five-digit numbers serving to identify the productions by Davidson. The data were reduced by the Jackson and Fleckenstein modification of the Morrissey-Gullikson method of computation,<sup>1/</sup> which evaluates the perceptual size of each difference on an interval scale with the zero of the scale placed at the average of the sizes of all 107 differences. To convert the interval scale to an additive scale, an additive constant was derived from the data obtained for the three double differences, as described by Judd<sup>3/</sup> in his paper entitled Interval Scales, Ratio Scales, and Additive Scales for the Sizes of Differences Perceived between Members of a Geodesic Series of Colors. The psychometric scale values of perceptual size so derived for each of the 107 differences are also listed in Table 3. The two hexagonal samples (maximum diameter of 50 mm) forming each difference were presented<sup>1/</sup> on an N 6/ background with a separation between them of about 8 mm. The identifying difference numbers used by the committee go up to 147. The sample numbers left blank in Table 3 refer to the samples listed for the previous number, but judged with no separation. The results for this condition were not used in the development of Munsell re-notations presented here.

The psychometric scale values of Table 3 were compared to the predictions of the four models summarized in Figs. 1 to 4, both in their original forms and after modifications intended to make them fit the experimental results better. The most successful modification was, for each model, a revision of the formula for the perceived size of color difference so as to count the hue component of the differences about twice as much as that corresponding to Euclidian geometry. Table 4 shows the correlation coefficients obtained for each model, both as originally proposed, and as optimized.

Table 5 summarizes the steps by which the Munsell model was changed in order to raise the correlation coefficient from 0.45 to 0.76. The correlation coefficients were obtained by a program written by Dr. G. L. Howett for a time-sharing, teletype access, GE computer at CEIR. Note that substitution of the Nickerson<sup>4/</sup> index of fading for the Godlove Euclidian formula in her paper entitled The Specification of Color Tolerances, raised the correlation coefficient from 0.45 to 0.61; and taking the square-root of the sum of the squares of the hue and chroma components of the chromaticness differences (instead of simply their sum as in the Nickerson index of fading) raised it from 0.61 to 0.65. The remainder of the improvement from 0.65 to 0.76 resulted from tentative redefining of the locus of colors corresponding to Munsell value 6/ and chroma /6, and the tentative respacing of the points on that locus defining the Munsell hue scale. Tables 6 and 7 define the 7th revision in terms of Munsell re-notations.

It is the purpose of the present report:

- (1) to describe further trials yielding still further improved agreement with the experimental data obtained by the OSA Committee on Uniform Color Scales,
- (2) to describe the extension of the redefinition of the 6/6-locus to higher and lower chromas at value 6/ and to values higher and lower than 6/ at all chromas, and
- (3) to specify the approximately 3,000 colors by which Munsell re-renotations may be defined.

This was done to yield maximum agreement with the spacing data so far developed by the OSA Committee on Uniform Color Scales at value 6/, with the Munsell renotations at values other than 6/, and with the results of other studies of color spacing described in the literature (Takasaki on chromatic crispening, Billmeyer on fanning out of the constant-hue loci in the red-purple to red hue range, Wyszecki and Sanders on colors of constant lightness, Evans on chroma scales). The spacing embodied in these re-renotations is supposed to appear uniform if, and only if, the two colors forming the difference are viewed on a gray background whose Munsell value approximates the average of the values of the two colors.

#### Further improvement of the 6/6 locus and hue scale

The method of making further improvements in the correlation between the Munsell model and the psychometric scale values obtained experimentally by the OSA Committee on Uniform Color Scales was the same as that used in the studies summarized in Table 5. The hue and chroma notations for each of the 43 test colors in the triangular array diagrammed in Fig. 5 were determined for each revision by plotting on the polar-coordinate system of renotation hue against renotation chroma the ovoid corresponding to the redefinition of the chroma scale (as in Table 6); then the revised loci for chromas /2, /4, /8, /10 and /12 were drawn in by linear interpolation and extrapolation of the chroma /6 locus; then the loci of revised hue were drawn in as forty straight lines passing through the neutral point and the point on the chroma /6 locus for that hue. The Munsell renotation hues and chromas for the 43 test colors were plotted from Table 2 on the original polar coordinates of the plot, and the revised hues and chromas were read for these points by interpolation on the revised chroma and hue loci. The hue and chroma components of the 107 differences evaluated by the Committee were then computed and fed into some form of formula for predicting the perceived sizes. The predictions of the model were then plotted against the psychometric scale values so as to identify, out of the 107 differences, the 10 or 12 color differences most overvalued by the model; then the 10 or 12 most undervalued. These differences would be marked on the lattice of Fig. 5 and inspected for any regularity of pattern. Alterations would then be made either in the chroma locus, the hue spacing, or the formula for computing the size of color difference of such nature as to tend to counteract the regularity. New predictions would then be computed and the process continued. This method is called the method of "outliers".

The first step of the improvement was to correct an error in our application of the Committee 1966 Munsell Formula (see Fig. 3):



$$dE = \left[ (C/5)(1.75 \Delta H) \right]^2 + (3\Delta C)^2 + (6\Delta V)^2 \quad 1/2 \quad (1)$$

Formula (1) is written in differential notation because it is strictly applicable only to hue difference  $\Delta H$  of infinitesimal size, but it is actually a good approximation for hue differences up to about 10 Munsell steps, or one-tenth of the hue circuit. In correlations 11, 12, 13 and 16 this formula had been applied to hue differences of all sizes, and then corrected by the ratio of the chord to the arc for the large hue differences. This correction is derived from, and is applicable to Euclidian geometry, as in the Godlove formula, but is in error by nearly a factor of 2 for the Committee 1966 formula. Note that for  $\Delta V = 0$ ,  $C_1 = C_2 = C$ ,  $\Delta H = 50$  corresponds to complementary hues, and the difference should be equivalent to that corresponding to  $\Delta C = 2C$ . Formula (1) gives for this case:

$$dE_{\Delta H=50} = (C/5)(1.75 \times 50) = 17.5C$$

If this be corrected by the ratio of the chord (2C) to the arc ( $\pi C$ ), we obtain:

$$\text{Corrected } dE_{\Delta H=50} = 17.5C(2/\pi) = 35C/\pi = 11.14C$$

This corrected amount should equal the calculated color difference corresponding to  $\Delta C = 2C$ , which is:

$$dE_{\Delta C=2C} = 6C$$

and it is seen that even the corrected value of  $dE_{\Delta H=50}$  is too high by a factor of  $11.14/6 = 1.86$ .

To avoid this error the Committee 1966 formula has been extended to apply to hue differences of all sizes from 0 to 50 Munsell hue steps and to any desired value of hue superweight factor,  $k$ , between 1 and 2. The extended formula may be written:

$$\Delta E = \left\{ \left[ f_g f_h (C_1 C_2)^{1/2} \Delta H \right]^2 + (\Delta C)^2 + (4\Delta V)^2 \right\}^{1/2} \quad (2)$$

where  $C_1$  is the Munsell chroma of the first color;  $C_2$ , that of the second;  $\Delta C$ , the chroma difference between the two;  $\Delta H$ , the Munsell hue difference on the 100-step scale;  $\Delta V$ , the difference in Munsell value;  $f_g$  is an abbreviation for the term:  $[2(1 - \cos 3.6^\circ \Delta H)]^{1/2} / \Delta H$ , required to convert the hue component of the Godlove formula to a term in  $\Delta H$ ; and

$$f_h = 2 - k + 4(k-1)/(3 - \cos 3.6^\circ \Delta H)$$

If the hue-superweight factor  $k$  is set equal to unity, it may be noted that  $f_h = 1$ ; and substitution of  $f_h = 1$  into formula (2) makes it identical to the Godlove formula. Note furthermore that substitution of  $\Delta H = 50$  into the definition for  $f_h$  likewise yields  $f_h = 1$ ; so the extended formula (2) yields:

$\Delta E_{\Delta H=50} = 2C = E_{\Delta C=2C}$ , as it should, regardless of the value assigned to the hue-superweight factor. Computation of size  $\Delta E_H$  of a hue difference for a value of  $\Delta H$  intermediate to 0 and 50, and for various values of the hue-superweight factor  $k$ , indicate that this extension of formula (1) is applicable for all values of  $\Delta H$  from 0 to 50, and for all values of  $k$  from 1 to 2. For  $k \approx 2$ , the extended formula indicates that  $\Delta E_H$  for  $\Delta H = 30$  is 1.966, only slightly less than that, 2.00C, for  $\Delta H = 50$ , which is believable. But for  $k$  greater than 2,  $\Delta E_H$  for  $\Delta H$  less than 50 can exceed that, 2.00C, for  $\Delta H = 50$  which must be rejected. We have used this formula for optimization of  $k$  between 1 and 2. Table 8 shows the product,  $f_g f_h$ , as a function of  $\Delta H$  between 0 and 50, for  $k = 1.5, 1.6, 1.7, 1.8, 1.9,$  and  $2.0$ .

A second step in the improvement of the correlation between the predictions of the Munsell model and the psychometric scale values obtained experimentally by the OSA Committee was the introduction of a term,  $f_s$ , intended to take account of the chromatic difference between the average of the two colors forming the difference and the neutral color of the background. Schönfelder<sup>5</sup>, in his paper, *Der Einfluss des Umfeldes auf die Sicherheit der Einstellung von Farbgleichungen*, long ago showed that the most favorable background color for the detection of the difference between two colors is the average of those two colors. Takasaki<sup>6</sup> just recently has showed in his paper entitled *Chromatic Changes Induced by Changes in Chromaticity of Background of Constant Lightness*, that chromatic induction is greatest for those colors most like the inducing color, and falls away with increasing departure of the test color from the inducing color. He called this phenomenon "chromatic crispening". In accord with these findings, it would be expected that a series of colors of constant lightness ranging from yellow through gray to the complementary blue and selected so as to appear equally spaced when viewed against a gray background of the same lightness should appear unequally spaced when viewed against a yellow background. The inequality of spacing should take the form of large perceived differences among the yellows of the scale closely resembling the background, and small perceived differences among the extreme blues of the scale. In an attempt to study whether this aspect of the perception of color differences is optimally incorporated into the Munsell model we have extended formula (2) by multiplying the right-hand term by the factor,  $f_s$ , thus:

$$\Delta E = f_s \left\{ \left[ f_g f_h (C_1 C_2)^{1/2} \Delta H \right]^2 + (\Delta C)^2 + (4\Delta V)^2 \right\}^{1/2} \quad (3)$$

where:  $f_s = (15 + \bar{C}) / (5 + \bar{C})$ ,  $\max f_s = 3$ , or

$$= (20 + \bar{C}) / (10 + \bar{C}), \max f_s = 2,$$

$\bar{C}$  being an abbreviation for the average chroma,  $(C_1 + C_2) / 2$ .

The other steps by which improvement in correlation was achieved were adjustments of the spacing of the points around the 6/6 locus defining the hue scale. Table 9 summarizes the steps by which the correlation coefficient was raised from 0.742 to 0.795. Application of the correct formula (2) instead of the wrongly corrected differential formula

(1) resulted in a lowering of the correlation coefficient from 0.761 to 0.754. The interpretation of this lowering is that a part of the high correlation indicated by the coefficient 0.761 is spurious and fortuitous. Although use of the correct formula yielded a lower coefficient of correlation, the resulting outliers showed a definite pattern and pointed the way to the 8th revision differing from the 7th only by a slightly revised hue scale. The 8th revision yielded a slight improvement in correlation coefficient to 0.762, and to a different regularity in the pattern of outliers pointing to a further adjustment of the hue scale. The 9th revision yielded a correlation coefficient of 0.795 for  $k = 1.6$  and  $f_s = 1$ . Table 10 gives the definition of the 9th revision hue scale; the chroma scales are the same as those for the 7th revision; see Table 6.

Step 18 indicated that max  $f_s$  equal to 3 was an overcorrection for the influence of the chromaticity (neutral) of the background. Step 21 indicated that max  $f_s$  equal to 2 was likewise a slight overcorrection, but the resulting pattern of outliers indicated that the undervaluation of the near-gray differences by the model for  $f_s = 1$  had been corrected. The reason for the lowered coefficient of correlation was that differences among the high-chroma colors also became undervalued relative to a group of chroma differences in the middle-chroma part of the green hue region. It was decided to stand with the 9th revision of the hue scale, and to try adjustments of the chroma scale in the green region at the time of extending the chroma scales above chroma /6.

Extension of value 6/ redefinition to chromas other than /6

It was the original plan to develop the chroma scales of the Atlas of the Munsell Color System by taking equal increments of area for the chromatic color in rotary mixtures with the neutral gray of the same Munsell value on the Maxwell disk. It is shown in a paper by Gibson and Nickerson<sup>7/</sup> entitled An Analysis of the Munsell Color System Based on Measurements Made in 1919 and 1926, that chroma scales developed in this way correspond to constant dominant wavelength and yield chroma C proportional to colorimetric purity,  $p_c$ . See their equation (GN 11):

$$C = V p_c(V/C) / p_c(5/5) \quad \text{(GN 11)}$$

where V is Munsell value,  $p_c(V/C)$  is colorimetric purity of the color of Munsell value V and Munsell chroma C, and  $p_c(5/5)$  is colorimetric purity of the 5/5 color of the same dominant wavelength. For any given dominant wavelength,  $p_c(5/5)$  is a constant; so for any dominant wavelength and any given value V, their equation (GN 11) implies a proportionality between Munsell chroma and colorimetric purity. Measurements made in 1919 and 1926 indicate rough agreement with this formula, particularly for middle Munsell values<sup>7/</sup>; but there are consistent deviations from them, greater at low Munsell values than at high. In the revision of these scales to form the Munsell Book of Color, the loci of constant hue were changed so

as to deviate considerably more from constant dominant wavelength, and the chroma scales were likewise somewhat changed. In a review of the spacing of the Munsell colors by Newhall<sup>8/</sup> in a paper entitled Preliminary Report of the O.S.A. Subcommittee on the Spacing of the Munsell Colors, further adjustments were made to improve the visual uniformity of the chroma scales. In spite of these three stages of visual adjustment, a recent analysis by Evans and Swenholt<sup>9/</sup> in their paper The Chromatic Strength of Colors, Part II; The Munsell System, has shown that the proportionality of Munsell chroma with colorimetric purity still holds to a surprising extent. It was therefore decided to extend tentatively the chroma scales, already determined, at value 6/ above and below chroma /6 by means of proportionality to colorimetric purity modified by a surround factor (max  $f_s = 2$ ), and to introduce other modifications of these scales only if required to conform either to the psychometric scale values of the OSA Committee or to well-established Munsell renotations.

The first step in deriving such tentative chroma scales was to read from the value 6/ chart defining the Munsell renotations as given by Newhall, Nickerson, and Judd<sup>10/</sup> in their paper entitled Final Report of the O.S.A. Subcommittee on the Spacing of the Munsell Colors, the values of (x,y) chromaticity coordinates corresponding to the Munsell renotations defining the 6/6 locus of the 9th revision from Tables 6 and 9. These values of chromaticity coordinates were plotted on the (x,y)-chromaticity diagram and slightly smoothed. Table 11 gives both the unsmoothed and smoothed values of chromaticity coordinates. The values of excitation purity,  $p_e$ , corresponding to the smoothed values of x and y were then read from the Hardy graphs<sup>11/</sup> in his Handbook of Colorimetry, and checked by computation. In this computation the chromaticity coordinates ( $x_b, y_b$ ) of the border (spectrum locus and line connecting its extremes) were read by interpolation in the tables given by Judd<sup>12/</sup> in his paper entitled The 1931 I.C.I. Standard Observer and Coordinate System for Colorimetry, from the slopes of the dominant (or complementary) wavelength lines:  $(y - 0.3163)/(x - 0.3101)$ . The values of excitation purity  $p_e$  were computed from the formula;

$$p_e = \frac{(x - 0.3101)/(x_b - 0.3101), \text{ or}}{(y - 0.3163)/(y_b - 0.3163)}, \quad (4)$$

whichever had the numerator of the greater absolute value. The coordinates (0.3101, 0.3163) are those of CIE source C. Finally the values of colorimetric purity,  $p_c$ , corresponding to the values of  $p_e$  were computed from Hardy's formula (reference 11, p. 60):

$$p_c = \frac{y_b p_e}{0.3163 + p_e (y_b - 0.3163)} \quad (5)$$

Table 11 also gives values of excitation purity,  $p_e$ , and colorimetric purity,  $p_c$ , for each of the forty points on the 6/6 locus defining the 9th revision, and values of the chromaticity coordinates ( $x_b, y_b$ ) of the border of the chromaticity diagram at the same dominant (or complementary) wavelength.

For any given dominant (or complementary) wavelength the approximate rule that Munsell chroma shall be made proportional to colorimetric purity may be written as:

$$p_{cC} = (p_{c6}/6) C \quad (6)$$

If we wish to make the chroma scale take into account the fact that the perceived size of a given difference increases by a factor  $f_s$  (max  $f_s = 2$ ) increasing as the average of the two colors making up the difference approaches the color of the surround, then each step in colorimetric purity should be decreased by a factor equal to  $1/f_s$ , and the selection of the colorimetric purity would conform to the law:

$$p_{cC} = p_{c6} \sum_0^C (K/f_s) \quad (7)$$

where  $K$  is a constant of proportionality adjusted so as to make  $p_{cC}$  for  $C = 6$  equal to  $p_{c6}$ , and  $\sum_0^C (K/f_s)$  is the sum of  $K/f_s$  from chroma equal to zero up to chroma equal to  $C$  in steps of 2. If the expression  $(20 + \bar{C}) / (10 + \bar{C})$ , as in formula (3), is taken to represent the variation of  $f_s$  with chroma, then this formula (7) may be written:

$$p_{cC} = K p_{c6} \sum_0^C (10 + \bar{C}) / (20 + \bar{C}) \quad (8)$$

Note, however, that the average chroma  $\bar{C}$  of the colors making up a chroma difference  $\Delta C = 2$ , is equal to  $C-1$ ; that is, the average chroma of the difference between  $C = 10$  and  $C = 8$ , for example, is 9. Formula (8) may thus take the form:

$$p_{cC} = K p_{c6} \sum_0^C (9 + C) / (19 + C) \quad (9)$$

Table 12 evaluates the summation indicated in formula (9), and shows the ratios,  $p_{cC}/p_{c6}$ , up to  $C = 38$ . It is evident from formula (9) that  $K = 1/\sum_0^6 (9 + C) / (19 + C)$ , for  $C = 6$ , and Table 12 shows this to be:  $1/1.689 = 0.5921$ . The final column shows for comparison the same ratio from the linear formula (6) by which  $p_{cC}/p_{c6} = C/6$ ,

To define the chroma loci at value 6/, the colorimetric purities for each of the 40 lines of constant dominant (or complementary) wavelength were computed by formula (9) corresponding to chromas /2, /4, /6, /8, ... until the next increment of 2 chroma steps produced a value of colorimetric purity exceeding 1.05. The values of excitation purity corresponding to these values of colorimetric purity were then computed by Hardy's formula<sup>10/</sup>:

$$p_e = \frac{0.3163 p_c}{y_b - p_c (y_b - 0.3163)} \quad (10)$$

Then the values of the chromaticity coordinates, x,y, for each of the points on the 40 lines of constant dominant (or complementary) wavelength were computed from the formulas:

$$\begin{aligned} x &= 0.3101 + p_e (x_b - 0.3101) \\ y &= 0.3163 + p_e (y_b - 0.3163) \end{aligned} \quad (11)$$

derived from formula (4).

The computations were carried out by the teletype-access GE computer at CEIR by means of a program written by Dr. G. T. Yonemura in accord with formulas (9), (10), and (11). This method yielded 40 points on each revised locus of constant chroma. These points were plotted on an (x,y)-chromaticity diagram, and joined by a smooth curve, to produce each locus of constant chroma.

The lines of constant revised Munsell hue were drawn as curves through the illuminant point,  $x = 0.3101$ ,  $y = 0.3163$ , and through each of the points on the 6/6 locus defining the hue scale of the 9th revision, the curvature used for each line generally being like that used for the Munsell renotations for the same part of the (x,y)-chromaticity diagram. An exception was made in the red-purple to red region (2.5R to 7.5R) in which the lines were made more nearly straight so as to conform to the observations reported by Billmeyer, Beasley, and Sheldon<sup>13/</sup> in their paper entitled Color-Order System Predicting Constant Hue.

Although an attempt has been made to duplicate approximately the 9th revision by using a 6/6 locus only slightly smoothed from that of the 9th revision, the use of formula (9) involving proportionality of chroma with colorimetric purity with a correction for the influence of the gray surround gives chroma scales somewhat different from those implied in the 9th revision by the use of linear interpolation and extrapolation of the Munsell renotation chroma scales for chromas different from /6. The revision embodied in the graph whose construction has just been described is therefore referred to as the 10th revision.

#### Improvement of the 10th revision

A check of the implications of the 10th revision was made by reading the Munsell re-renotations of the 43 test colors studied by the OSA Committee, and computing the sizes predicted for the color differences of Table 3. It was found that the correlation coefficient had dropped to 0.734 for the 10th revision from 0.795 for the 9th revision as shown in Table 9. A plot of the outliers indicated that formula (9), based on a close relationship of chroma with colorimetric purity, had yielded chroma

scales that grossly undervalued the differences in the high-chroma portions of the green region.

As a partial correction, an 11th revision was produced by altering the chroma spacing between 5R and 5BG above chroma /6 so that the step on the (x,y)-diagram along any line of constant dominant wavelength corresponding to an increment of 2 chroma steps was held constant at the separation corresponding to the step from /4 to /6 chroma. This was not sufficient. Therefore, a 12th revision was produced in which the separation equals the step from /2 to /4 chroma (instead of 4 to 6 chroma). Table 13 summarizes the coefficients of correlation obtained by the 10th, 11th, and 12th revisions.

Note from Table 13 that the formula used to obtain these correlations is formula (3) with  $f_s$  set equal to unity which is equivalent to formula (2). It should be recalled, however, that the spacing has been adjusted to a correction for the influence of the surround equivalent to  $\max f_s = 2$ . The highest correlation coefficient obtained (0.803 for the 12th revision with a super-hueweight factor,  $k = 1.7$ ) seems to be close to the highest obtainable by any believable redefinition of the scales of a Munsell type model, and even this requires a rather marked departure from linearity in the chroma scales in the green hue region which is hard to believe; that is, the distance on the (x,y)-diagram for the /0 to /2 interval is the smallest, that for /2 to /4 somewhat larger, that for /4 to /6 still larger, and then the remainder of the steps revert to the distance on the (x,y)-chromaticity diagram for the /2 to /4 step. The reason for inclusion of this hard-to-believe feature is that the experimental data developed by the OSA Committee clearly indicate it. It should be recalled that the principal purpose of this study is to develop Munsell re-renotations yielding maximum agreement with those spacing data.

Table 14 gives the Munsell re-renotation hue and chroma of the 43 test colors studied by the OSA Committee, also the differences from renotation hue and chroma. Table 15 gives the steps by which the perceived sizes of the 107 color differences predicted by the 12th revision are found by formula (2). Column (9) gives the predictions which yield a correlation coefficient of 0.803 with the psychometric scale values listed in Table 3.

Extension to Munsell values above and below 6/

The extension of the definition of the chroma loci for Munsell values above and below 6/ was based, after checking on the exponent for best fit to the renotations, on multiplying the colorimetric purities defining the 6/6 locus by the cube-root of the inverse ratio of the reflectances Y corresponding to the Munsell value  $\frac{10}{6}$ , thus:

$$P_{cV/6} = P_{c6/6} \frac{Y_{V=6}^{1/3}}{Y_V^{1/3}} \quad (12)$$

Table 16 shows the computation of the ratios for the values 1/ to 10/.

Each of the 40 values of colorimetric purity  $p_c$  defining the 6/6 locus, listed in Table 11 were multiplied by each of these nine ratios to yield definitions of the chroma /6 locus for Munsell values 1/, 2/, 3/, 4/, 5/, 7/, 8/, 9/, and 10/. The extension to other chromas of these definitions of the chroma /6 loci for each value level was then carried out as described for value 6/.

The loci of constant hue were drawn in by analogy to the loci of constant Munsell renotation hue at each value level as described previously for value 6/. The fanning out of the constant hue lines from one value level to another likewise was done by analogy to the fanning out of the constant renotation hue lines except for the 2.5RP to 10R range. In this range the fanning out was adjusted so as to conform to the observations reported by Billmeyer, Beasley, and Sheldon<sup>13/</sup>. The result of this adjustment was to eliminate the extreme fanning out characterizing the Munsell renotation hue lines in this hue region, and to reduce it about to the average fanning out of the Munsell renotation hue lines for the hue regions other than 2.5RP to 10R.

The hue and chroma loci derived in this way were plotted on large sheets of mat-finish plastic (K & E Herculene), and their intersections were read to yield values of (x,y)-chromaticity coordinates by which the Munsell re-notation hue and chroma scales may be tentatively defined in a way analogous to the definition of the Munsell renotation hue and chroma scales by Newhall, Nickerson, and Judd.<sup>10/</sup> The resulting chroma scales were reviewed and were found to be not too much different from the Munsell renotation chroma scales within the limits of the existing paint gamut, except for the high-chroma range (/10 to /20) in the purple-blue to purple hue range, where the chroma loci on the (x,y)-diagram become increasingly close together. The tentative re-notation chroma scales show this property to about twice the extent of the renotations. As an example the Munsell renotations of the 7.5PB 3/ chroma scale are listed in Table 16, together with the corresponding chromaticity coordinates, x,y, for CIE source C from reference 10. From these chromaticity coordinates the corresponding tentative re-notation chromas were found from the large graph for value 3/ to be as shown in Table 17. A comparison of the re-notation chromas listed column 4 with the renotation chromas listed in column 1 shows that re-notation chroma rises at high chromas about twice as fast as renotation chromas which is unbelievable. As a result of comparisons of this sort, the tentative re-notation chroma scales were accepted for the blue and purple hue regions (5BG to 5R) only up to chroma /10. Thereafter, the (x,y)-interval corresponding to the chroma difference between /8 and /10, was used for chromas /12, /14, and so on. Table 17 shows the chroma assignments resulting from the re-renotations developed in this way.

#### Derivation of reflectances Y defining Munsell re-notation value

The reflectances Y defining the Munsell re-notation value scale for the neutral colors N 0/ through N 10/ were taken equal to those for the Munsell renotation value scale. For chromatic colors (chroma greater than zero), these values of reflectance were decreased to conform to the results of the auxiliary study by the OSA Committee on Uniform Color Scales already



mentioned; see reference 2. The reflectance Y for each of the approximately 3,000 colors defining the Munsell re-notations was computed by the formula:

$$Y = (Y_{Vr}/L) Y_{Vr} \quad (13)$$

where  $Y_{Vr}$  is the reflectance of any sample of value V according to the Munsell-notation value function, and L is the reflectance of the gray sample which presumably would have been found by the OSA Committee to appear equally light as the chromatic sample. Note that if  $Y_{Vr}$  refers to a gray sample, the ratio  $Y_{Vr}/L = 1$ ; so for grays  $Y = Y_{Vr}$  as intended. For all other samples, that is, chromatic samples, the reflectance Y assigned to a given Munsell re-notation value will be somewhat less than  $Y_{Vr}$ .

The method of evaluating Y for the chromatic samples was to plot on the (x,y)-chromaticity diagram the locus of colors found by Wyszecki<sup>2</sup> to correspond to the ratio:  $L/Y_{Vr} = 1.35$ . Loci for other values of  $L/Y_{Vr}$  (0.05, 0.10, 0.15, ... 2.20) were found by linear interpolation and extrapolation of the 1.35 locus on the (x,y)-diagram. The values of the ratio,  $L/Y_{Vr}$  for each of the approximately 3,000 chromaticity points defining the Munsell re-notation hue and chroma scales were read by plotting the point on the (x,y)-diagram and interpolating among the loci plotted there. The values of reflectance Y defining the Munsell re-notation value scales for chromatic colors were found by dividing the value of  $Y_{Vr}$  by the value of the ratio:  $L/Y_{Vr}$ , in accord with formula (13).

Table 18 gives the definition of the Munsell re-notation hue, value and chroma scales so found. A count of the entries in this table indicates that there are 2,874.

## Summary

Munsell re-renotations have been developed from experimental findings since 1943 both by the OSA Committee on Uniform Color Scales and by other studies appearing in the literature as follows:

1. The 6/6 chroma locus, and hue spacing there, were redefined to agree with the experimental results by the OSA Committee.
2. The extension at value 6/ to chromas other than /6 was by a combination of proportionality to colorimetric purity,  $p_c$ , with the Takasaki crispening factor,  $f_s$  ( $\max f_s = 2$ ), with two exceptions:
  - a. In the green hue region (5R to 5BG) above chroma /6 the chroma scales were made linear in excitation purity by applying the (x,y)-distance corresponding at constant dominant wavelength to the distance between chroma /2 and /4 to achieve agreement with the results of the OSA Committee.
  - b. In the purple hue region (5BG to 5R) the chromas above /10 were similarly made linear with excitation purity by continuing the (x,y)-distance corresponding to the two-step chroma interval from /8 to /10 to make the spacing correspond more closely to the Munsell renotations.
3. The extension to values other than /6 was by the inverse ratio of the cube-roots of the reflectance defining the Munsell renotations.
4. The loci of constant hue were drawn in with the curvature generally similar to that exhibited by the Munsell renotation constant-hue loci in the same part of the (x,y)-diagram with two exceptions:
  - a. The curvature in the red hue region was reduced so as to agree with the experimental results of Billmeyer et al.<sup>13/</sup>
  - b. The fanning out of the constant hue loci with Munsell value was reduced in the red region so as to conform to the same experimental results.<sup>13/</sup>
5. Formula (2) was used for computing the predicted sizes of color differences. Formula (2) is a modification of the Godlove Euclidian formula in which the super-hueweight factor,  $k$ , is set equal to 1.7 to achieve maximum agreement with the experimental results of the OSA Committee.
6. The assignment of reflectances Y to the neutral scale agrees precisely with the Munsell renotation value scale, but for chromatic colors the assignment was such as to make colors of constant Munsell re-renotation value appear equally light in accord with Wyszecki's reduction of the experimental results obtained by the OSA Committee.
7. These adjustments of the Munsell renotation scales to make the predictions of the perceived size of a color difference accord with formula (2) apply to appraisals of the color-difference made with a neutral background of the same Munsell re-renotation value as the average of the colors making up the difference.

## Discussion

This set of re-renotations should not be regarded as final. Steps 2 and 3, particularly, are little more than informed guesses. The study of color differences involving about equally large lightness and chromaticness components now under way by the OSA Committee will probably yield discrepancies with the re-renotations that will have to be taken into account; and, in general, considerably more experimental work should be done before a final set of re-renotations can be recommended. To paint up an atlas of color on the basis of this set of re-renotations might be an economical way to start this experimental work.

Two questionable features of the Munsell renotations have been eliminated in this set of re-renotations.

1. The off-set of the chroma loci as value shifts from one level to another has been given up.

2. The use of the same chroma loci for value  $/10$  as for value  $/9$  has likewise been given up. *10/9*

The arbitrary departure from the use of the law of proportionality between chroma and colorimetric purity for chromas above  $/10$  for the purple-blue and purple hue regions cannot be wholly correct. If the Munsell system were to be defined in terms of the 1964 CIE supplementary observer, the extreme crowding of the constant-chroma loci about  $/10$  would be avoided, and if this should turn out to be not correct a formula for the spacing of the chroma loci such as to avoid a discontinuity in spacing at chroma  $/10$  could easily be devised. The discontinuity in spacing introduced in the green hue region to conform to the findings of the OSA Committee, however, poses a much more difficult problem of formulation though a formula still might be written to give a smooth transition from the colorimetric-purity basis for low chromas to the excitation-purity basis for high that we have used. One should probably be skeptical of the reality of the experimental finding that seems to indicate that the spacing of the chroma loci in the green region should increase up to chroma  $/6$  and then decrease from  $/6$  to  $/8$ .

The fact that two arbitrary departures had to be made from proportionality between Munsell chroma and colorimetric purity indicates that this law is only approximate. The development of chroma loci in accord with this law modified by the surround factor,  $f_s$ , however, was not wasted effort. The two departures (one for the green hue region above chroma  $/6$ , one for the purple hue region above  $/10$ ) could be made smooth by making the transition at the hues corresponding closely to  $y = y_w = 0.3163$  where colorimetric and excitation purity are identical.

References

- 1/ D. B. Judd, Farbe 14, 287 (1965).
- 2/ Günter Wyszecki, J. Opt. Soc. Am. 57, 254 (1967).
- 3/ D. B. Judd, J. Opt. Soc. Am. 57, 380 (1967).
- 4/ D. Nickerson, Textile Research, 6, 509 (1936).
- 5/ W. Schönfelder, Z. Sinnesphysiol, 63, 228 (1933).
- 6/ H. Takasaki, J. Opt. Soc. Am. 57, 93 (1967).
- 7/ K. S. Gibson, and D. Nickerson, J. Opt. Soc. Am. 30, 591 (1940).
- 8/ S. M. Newhall, J. Opt. Soc. Am. 30, 617 (1940).
- 9/ R. M. Evans and R. K. Swenholt, submitted in 1967 for publication in J. Opt. Soc. Am.
- 10/ S. M. Newhall, D. Nickerson, and D. B. Judd, J. Opt. Soc. Am. 33, 385 (1943).
- 11/ A. C. Hardy, The Technology Press, Massachusetts Institute of Technology, Cambridge, Massachusetts, pp. 60-84.
- 12/ D. B. Judd, J. Opt. Soc. Am. 23, 359 (1933).
- 13/ F. W. Billmeyer, Jr., J. K. Beasley, and J. A. Sheldon, J. Opt. Soc. Am. 51, 656, 1440 (1961).

Table 1: Some details of the four preliminary checks of color spacing

Dates	Purpose	No. of Colors	No. of diffs.	No. of pairs of diffs. shown	No. of obser- vers	No. of obser- vations	Data reduced by:
May, 1955 to June, 1956	Constant saturation locus at value 6/	34	33	384	60	23,040	Nickerson, Judd, and Nimeroff
Sept., 1956 to Oct., 1957	Hue spacing on this locus	40	39	142	102	14,484	Newhall
Dec., 1958 to Sept., 1961	Chromaticness spacing for near grays of Munsell value 3/, 6/, and 8/	21	36	99	97	9,603	Howett
Jan., 1964 to	Chromaticness spacing for Munsell value 6/	43	147	436	70	9,940	Howett

Table 2, - Chromaticity coordinates (x,y), reflectances(Y) for CIE source C, and the corresponding Munsell rennotations (H V/C) for the 43 test samples of the fourth basic experiment by the OSA Committee on Uniform Color Scales

Sample No.	CIE Coordinates			Munsell rennotation
	Y	x	y	H V/C
1	30.99	0.2955	0.5321	0.4G 6.08/12.4
2	30.24	.2816	.4652	1.7G 6.02/10.0
3	30.08	.3341	.4813	8.2GY 6.00/8.5
4	29.08	.3777	.4976	5.2GY 5.92/8.5
5	29.22	.2520	.4038	5.6G 5.93/8.8
6	28.25	.2922	.4238	1.7G 5.84/7.2
7	28.14	.3439	.4416	6.8GY 5.83/6.2
8	26.20	.3838	.4466	1.9GY 5.65/5.8
9	28.53	.4379	.4599	5.7Y 5.86/7.7
10	28.59	.2299	.3383	4.0BG 5.87/7.6
11	28.16	.2661	.3603	8.4G 5.83/5.5
12	28.00	.3075	.3801	0.7G 5.82/4.0
13	29.59	.3544	.3982	2.4GY 5.96/3.7
14	28.75	.3959	.4100	4.7Y 5.88/4.8
15	28.91	.4415	.4193	0.6Y 5.90/7.0
16	28.54	.4764	.4238	8.6YR 5.87/9.0
17	28.71	.2258	.2926	1.3B 5.88/6.5
18	28.85	.2474	.3082	9.2BG 5.90/4.9
19	28.35	.2804	.3248	4.4BG 5.85/2.6
20	29.26	.3196	.3448	6.3GY 5.93/1.4
21	29.58	.3581	.3599	1.5Y 5.96/2.5
22	29.42	.4026	.3765	7.8YR 5.94/4.8
23	29.74	.4426	.3827	4.9YR 5.97/7.1
24	28.12	.4888	.3934	4.1YR 5.83/9.6
25	28.28	.2342	.2595	0.1PB 5.84/6.4
26	29.21	.2632	.2796	1.3PB 5.93/4.2
27	30.24	.2978	.3023	5.9PB 6.02/1.5
28	28.42	.3310	.3211	7.5R 5.86/1.5
29	29.68	.3713	.3355	9.7R 5.97/3.7
30	29.68	.4068	.3488	0.6YR 5.97/5.7
31	29.19	.4580	.3571	10.0R 5.93/8.6
32	30.09	.4864	.3602	9.7R 6.00/10.4
33	29.66	.2512	.2400	7.0PB 5.96/7.4
34	30.25	.2790	.2626	9.9PB 6.02/5.1
35	29.09	.3058	.2796	6.4P 5.92/4.1
36	29.54	.3398	.2994	5.3RP 5.96/4.2
37	28.86	.3785	.3157	2.5R 5.90/5.6
38	29.60	.4208	.3240	4.8R 5.96/8.2
39	29.40	.4580	.3288	5.6R 5.94/10.3
40	29.50	.2960	.2432	5.7P 5.95/8.1
41	28.74	.3258	.2576	10.0P 5.86/7.9
42	28.84	.3642	.2708	4.1RP 5.89/9.2
43	28.34	.3919	.2790	6.5RP 5.85/10.2

Table 3, - Identification of the test samples forming the 107 color differences evaluated, and the psychometric scale values,  $\bar{T}$ , derived for them by the linear model ( $K = 1.694$ ).

Diff. No.	Sample Nos.	T	Diff. No.	Sample Nos.	T	Diff. No.	Sample Nos.	T
1	1-2	1.727	51	14-21	1.262	101		
2	1-3	1.950	52			102	27-35	1.878
3	2-3	1.626	53	14-22	1.787	103	28-29	1.505
4	2-5	1.614	54	15-16	1.300	104	28-35	2.040
5	2-6	1.359	55			105	28-36	1.463
6			56	15-22	1.728	106		
7*	2-12	2.309	57	15-23	1.821	107	29-30	1.023
8	3-4	1.920	58			108		
9	3-6	2.084	59	16-23	1.730	109	29-36	1.548
10	3-7	1.523	60	16-24	1.897	110	29-37	1.118
11	4-7	1.403	61	17-18	1.277	111	30-31	1.534
12			62	17-25	2.540	112		
13	4-8	1.590	63	18-19	1.679	113	30-37	1.856
14			64	18-25	2.500	114		
15	5-6	1.695	65			115	30-38	1.524
16	5-10	1.945	66	18-26	2.409	116		
17	5-11	1.305	67	19-20	1.750	117	31-32	0.910
18	6-7	1.524	68	19-26	2.256	118	31-38	1.883
19			69			119	31-39	1.859
20	6-11	1.535	70	19-27	1.934	120	32-39	1.916
21	6-12	0.927	71			121		
22	7-8	1.670	72	20-21	1.687	122	33-34	2.025
23	7-12	1.166	73	20-27	1.838	123		
24			74	20-28	1.784	124	34-35	1.349
25	7-13	1.463	75			125	34-40	1.820
26	8-9	2.200	76	21-22	1.195	126		
27	8-13	1.275	77	21-28	1.339	127	35-36	1.412
28			78			128	35-40	1.607
29	8-14	1.887	79	21-29	1.762	129	35-41	1.773
30			80			130		
31	9-14	1.594	81	22-23	1.053	131	36-37	1.431
32	9-15	2.038	82			132		
33	10-11	1.274	83	22-29	1.228	133	36-41	2.132
34	10-17	1.896	84	22-30	1.614	134	36-42	1.513
35	10-18	1.627	85	23-24	1.380	135	37-38	1.113
36	11-12	1.586	86	23-30	1.399	136*	37-39	2.308
37			87	23-31	1.874	137	37-42	1.821
38	11-18	2.039	88	24-31	2.173	138	37-43	1.887
39	11-19	1.237	89			139	38-39	1.089
40	12-13	1.452	90	24-32	2.223	140		
41			91	25-26	1.668	141	38-43	2.201
42	12-19	1.703	92			142		
43			93	25-33	2.533	143	40-41	1.676
44	12-20	1.474	94			144	41-42	1.564
45	13-14	1.586	95	26-27	1.589	145		
46			96	26-33	2.243	146*	41-43	2.389
47	13-20	1.390	97	26-34	2.048	147	42-43	0.953
48			98					
49	13-21	1.490	99	27-28	1.495			
50	14-15	1.840	100	27-34	2.043			

Table 4, - Correlation coefficients between the sizes of 107 color differences predicted by four models of color spacing and the psychometric scale values found for those differences by the OSA Committee on Uniform Color Scales

	U*V*W*	Cube-Root	Munsell	MacAdam-Friele
Original unadjusted	0.23	0.34	0.45	0.39
Optimal adjustment as of April 1967 to the data obtained experimentally on the 107 differences	0.62	0.62	0.76	0.74 <sub>5</sub>



Table 5, - Summary of the steps by which the Munsell model was changed in order to raise the correlation coefficient from 0.45 (Munsell renotations with Godlove Euclidian formula) to 0.76(7th revision with Committee 1966 formula shown in Fig. 3).

No.	Spacing	Date	Measure	Correlation coefficient	Standard error of estimate % of mean
1	Munsell renotation	11/8/66	Distance on H-C diagram, Godlove formula	0.451	19.4
2	"	"	Nickerson Index of Fading	.609	17.3
3	"	11/10/66	Root-sum-square with hue weight as in Index of Fading	.649	16.5
4	2nd rev.	"	Distance on H $\ominus$ C diagram	.503	18.8
5	"	"	Index of Fading	.568	17.9
6	"	"	Root-sum-square with hue weight as in Index of Fading	.591	17.5
7	3rd rev.	11/18/66	"	.725	15.0
8	"	"	Same as above, but with hue weight reduced to 2/3 that in the Index of Fading (Hue weight = 2I/3)	.736	14.7
13	"	12/14/66	Hue weight = 3I/4	.744	14.5
14	"	"	Hue weight = 7I/8	.734	14.8
9	5th rev.	"	Hue weight = I	.695	15.6
10	"	"	Hue weight = 2I/3	.688	15.8
11	"	"	Hue weight = 3I/4	.706	15.4
12	6th rev.	"	Hue weight = 3I/4	.701	15.5
15	7th rev.	12/21/66	Hue weight = I	.732	14.8
16	"	"	Hue weight = 3I/4	.761	14.1

Table 6, - Definition of the 7th revision chroma /6 locus  
in terms of Munsell renotation chroma

Munsell renotation hue	Munsell renotation chroma for the colors of the 6/ value, /6 chroma locus of the 7th revision	Difference from /6
5R	7.0	1.0
10R	6.0	0.0
5YR	5.5	-.5
10YR	5.0	-1.0
5Y	5.0	-1.0
10Y	5.1	-.9
5GY	5.5	-.5
10GY	6.2	.2
5G	6.9	.9
10G	7.0	1.0
5BG	6.8	.8
10BG	6.0	.0
5B	5.5	-.5
10B	5.3	-.7
5PB	5.6	-.4
10PB	6.2	.2
5P	7.0	1.0
10P	8.2	2.2
5RP	9.0	3.0
10RP	8.3	2.3

Table 7, - Definition of the 7th revision hue scale in terms of Munsell renotation hues

Hue notation in the 7th revision	Munsell renotation of the corresponding hue	Renotation hue interval corresponding to each step of 2.5 in 7th revision
2.5R	1.8R	3.0
5.0R	4.8R	3.0
7.5R	7.8R	2.9
10.0R	0.7YR	2.7
2.5YR	3.4YR	2.6
5.0YR	6.0YR	2.4
7.5YR	8.4YR	2.3
10.0YR	0.7Y	2.2
2.5Y	2.9Y	2.3
5.0Y	5.2Y	2.3
7.5Y	7.5Y	2.3
10.0Y	9.8Y	2.4
2.5GY	2.2GY	2.4
5.0GY	4.6GY	2.4
7.5GY	7.0GY	2.3
10.0GY	9.3GY	2.3
2.5G	1.6G	2.2
5.0G	3.8G	2.3
7.5G	6.1G	2.4
10.0G	8.5G	2.9
2.5BG	1.4BG	3.2
5.0BG	4.6BG	3.2
7.5BG	7.8BG	3.0
10.0BG	0.8B	3.0
2.5B	3.8B	2.8
5.0B	6.6B	2.7
7.5B	9.3B	2.6
10.0B	1.9PB	2.4
2.5PB	4.3PB	2.2
5.0PB	6.5PB	2.1
7.5PB	8.6PB	2.0
10.0PB	0.6P	2.0
2.5P	2.6P	2.0
5.0P	4.6P	2.0
7.5P	6.6P	2.1
10.0P	8.7P	2.2
2.5RP	0.9RP	2.4
5.0RP	3.3RP	2.6
7.5RP	5.9RP	2.9
10.0RP	8.8RP	3.0
2.5R	1.8R	

Table 8, - Values of the product,  $f_g f_h$ , for  $\Delta H$  between 1 and 50  
and for six values of the hue-superweight factor  $k$

$\Delta H$	$k=1.5$	$k=1.6$	$k=1.7$	$k=1.8$	$k=1.9$	$k=2.0$
1	0.094	0.100	0.107	0.113	0.119	0.126
2	.094	.100	.106	.113	.119	.125
3	.094	.100	.106	.112	.118	.124
4	.093	.099	.105	.111	.117	.123
5	.092	.098	.104	.110	.116	.122
6	.091	.097	.103	.109	.115	.120
7	.091	.096	.102	.108	.113	.119
8	.090	.095	.101	.106	.112	.118
9	.089	.094	.100	.105	.110	.115
10	.087	.092	.098	.103	.108	.113
11	.086	.091	.096	.101	.106	.111
12	.085	.089	.094	.099	.103	.108
13	.083	.088	.092	.097	.101	.106
14	.082	.086	.090	.094	.099	.103
15	.081	.085	.088	.092	.096	.101
16	.079	.083	.086	.090	.094	.098
17	.078	.081	.085	.088	.092	.095
18	.076	.080	.083	.086	.089	.093
19	.075	.078	.081	.084	.087	.090
20	.073	.076	.079	.082	.084	.087
21	.072	.074	.077	.080	.082	.085
22	.070	.073	.075	.078	.080	.083
23	.069	.071	.073	.076	.078	.080
24	.067	.069	.071	.074	.076	.078
25	.066	.068	.070	.072	.073	.075
26	.065	.067	.068	.070	.072	.073
27	.063	.065	.066	.068	.070	.071
28	.062	.063	.065	.066	.068	.069
29	.061	.062	.063	.065	.066	.067
30	.060	.061	.062	.063	.064	.065
31	.058	.059	.060	.061	.062	.063
32	.057	.058	.059	.060	.061	.062
33	.056	.057	.058	.058	.059	.060
34	.055	.056	.056	.057	.058	.058
35	.054	.054	.055	.056	.056	.057
36	.053	.053	.054	.054	.055	.055
37	.052	.052	.053	.053	.053	.054
38	.051	.051	.051	.052	.052	.053
39	.050	.050	.050	.051	.051	.051
40	.049	.049	.049	.049	.050	.050
41	.048	.048	.048	.048	.048	.049
42	.047	.047	.047	.047	.047	.048
43	.046	.046	.046	.046	.046	.047
44	.045	.045	.045	.045	.045	.046
45	.044	.044	.044	.044	.044	.044
46	.043	.043	.043	.043	.043	.043
47	.043	.043	.043	.043	.043	.043
48	.042	.042	.042	.042	.042	.042
49	.041	.041	.041	.041	.041	.041
50	.040	.040	.040	.040	.040	.040

Table 9, - Summary of the steps by which the Munsell model was changed in order to raise the correlation coefficient from 0.76 to 0.79 (9th revision with formula 3,  $k = 1.6$ ,  $f_s = 1$ ).

Step No.	Revision of Munsell Spacing	Date	$k$	Max $f_s$	Correlation coefficient	Standard error of estimate, % of mean
15	7th	12/21/66	1.12	1	0.732	14.8
16	"	"	1.59	1	.761	14.1
-----						
17	7th	7/31/67	2.0	1	.742	14.6
18	"	"	2.0	3	.708	15.4
19	"	8/1/67	1.6	1	.754	14.3
20	"	"	1.5	1	.744	14.5
21	"	"	1.6	2	.745	14.5
22	"	"	1.6	2	.636	16.8
(Sum of hue and chroma components without squaring)						
23	8th	8/29/67	1.6	1	.765	14.0
24	"	"	1.5	1	.762	14.1
25	9th	8/31/67	1.6	1	.795	13.2
26	"	"	1.5	1	.783	13.5

Table 10. Definition of 9th revision hue scale in terms of Munsell Renotation Hues

(9th revision chroma /6 locus same as 7th revision; see Table 6)

<u>Hue notation in the 9th revision</u>	<u>Munsell renotation of the corresponding hue</u>	<u>Renotation hue interval corresponding to each step of 2.5 in 9th revision</u>
2.5R	9.5RP	2.8
5.0R	2.3R	3.0
7.5R	5.3R	3.2
10.0R	8.5R	3.2
2.5YR	1.7YR	3.2
5.0YR	4.9YR	3.0
7.5YR	7.9YR	2.5
10.0YR	0.4Y	2.3
2.5Y	2.7Y	2.3
5.0Y	5.0Y	2.6
7.5Y	7.6Y	2.8
10.0Y	0.4GY	2.8
2.5GY	3.2GY	2.6
5.0GY	5.8GY	2.3
7.5GY	8.1GY	2.2
10.0GY	0.3G	2.2
2.5G	2.5G	2.3
5.0G	4.8G	2.7
7.5G	7.5G	3.0
10.0G	0.5BG	3.0
2.5BG	3.5BG	3.0
5.0BG	6.5BG	3.0
7.5BG	9.5BG	2.8
10.0BG	2.3B	2.8
2.5B	5.1B	2.7
5.0B	7.8B	2.6
7.5B	0.4PB	2.4
10.0B	2.8PB	2.3
2.5PB	5.1PB	2.1
5.0PB	7.2PB	2.0
7.5PB	9.2PB	1.9
10.0PB	1.1P	1.9
2.5P	3.0P	1.9
5.0P	4.9P	1.8
7.5P	6.7P	1.8
10.0P	8.5P	1.8
2.5RP	0.3RP	1.9
5.0RP	2.2RP	2.0
7.5RP	4.2RP	2.5
10.0RP	6.7RP	2.8
2.5R	9.5RP	

Table 11. - Unsmoothed and smoothed values of the chromaticity coordinates (x,y) of forty points defining the 9th revision; also excitation purity,  $p_e$ , and colorimetric purity,  $p_c$ , of these points and the chromaticity coordinates of the border points ( $x_b, y_b$ ) of the same dominant (or complementary) wavelength.

9th Rev. hue at 6/6	Chromaticity Unsmoothed		Coordinates Smoothed		Purity		Chromaticity Coordinates	
	x	y	x	y	$p_e$	$p_c$	$x_b$	$y_b$
2.5R	0.391	0.301	0.3909	0.3018	0.2096	0.1717	0.6956	0.2472
5.0R	.398	.313	.3982	.3132	.2276	.2198	.6972	.3027
7.5R	.404	.326	.4040	.3250	.2749	.2944	.6517	.3480
10.0R	.409	.340	.4094	.3401	.3299	.3765	.6111	.3882
2.5YR	.412	.353	.4119	.3531	.3715	.4370	.5841	.4153
5.0YR	.413	.369	.4121	.3688	.4140	.4970	.5565	.4427
7.5YR	.411	.382	.4109	.3822	.4470	.5422	.5356	.4636
10.0YR	.406	.393	.4070	.3931	.4661	.5705	.5180	.4811
2.5Y	.402	.404	.4019	.4038	.4809	.5931	.5010	.4980
5.0Y	.396	.412	.3962	.4118	.4875	.6064	.4868	.5122
7.5Y	.390	.419	.3891	.4184	.4862	.6116	.4727	.5263
10.0Y	.380	.425	.3800	.4250	.4793	.6125	.4558	.5431
2.5GY	.367	.429	.3671	.4289	.4555	.5984	.4352	.5635
5.0GY	.351	.430	.3512	.4301	.4165	.5709	.4088	.5895
7.5GY	.330	.427	.3298	.4272	.3520	.5203	.3660	.6314
10.0GY	.309	.419	.3086	.4193	.2760	.4539	.3046	.6895
2.5G	.287	.404	.2865	.4042	.1863	.3633	.1834	.7882
5.0G	.270	.389	.2704	.3880	.1454	.3033	.0371	.8093
7.5G	.258	.374	.2580	.3742	.1702	.2987	.0040	.6567
10.0G	.248	.358	.2471	.3581	.2101	.3023	.0103	.5152
2.5BG	.240	.340	.2382	.3406	.2508	.3043	.0234	.4132
5.0BG	.234	.321	.2338	.3208	.2800	.2902	.0376	.3325
7.5BG	.232	.305	.2322	.3051	.2984	.2727	.0490	.2788
10.0BG	.234	.293	.2342	.2925	.3021	.2453	.0589	.2375
2.5B	.238	.282	.2388	.2830	.2945	.2115	.0680	.2032
5.0B	.243	.275	.2432	.2749	.2875	.1802	.0774	.1723
7.5B	.248	.269	.2482	.2689	.2771	.1497	.0867	.1453
10.0B	.253	.264	.2529	.2644	.2665	.1225	.0955	.1215
2.5PB	.257	.260	.2575	.2610	.2553	.0945	.1041	.0966
5.0PB	.264	.256	.2644	.2562	.2400	.0617	.1197	.0659
7.5PB	.270	.252	.2719	.2529	.2247	.0303	.1401	.0341
10.0PB	.277	.250	.2779	.2510	.2159	.0119 <sub>5</sub>	.1610	.0139
2.5P	.284	.250	.2840	.2492	.2216	.0120	.1923	.0135
5.0P	.292	.250	.2925	.2488	.2391	.0327	.2365	.0340
7.5P	.304	.250	.3028	.2490	.2575	.0569	.2818	.0550
10.0P	.316	.253	.3169	.2513	.2747	.0871	.3349	.0797
2.5RP	.329	.257	.3295	.2552	.2826	.1108	.3788	.1001
5.0RP	.344	.263	.3442	.2620	.2827	.1340	.4307	.1242
7.5RP	.362	.273	.3622	.2727	.2732	.1570	.5008	.1567
10.0RP	.379	.286	.3789	.2860	.2508	.1714	.5844	.1955

Table 12. - Numerical evaluation of  $p_{cC}/p_{c6}$  from formula (9) for chroma in steps of 2 from 2 to 38.

Chroma <u>C</u>	<u>9 + C</u>	<u>19 + C</u>	<u>1/f<sub>s</sub></u>	<u><math>\sum_{0}^C (1/f_s)</math></u>	<u><math>P_{cC}/P_{c6}</math></u>	<u>C/6</u>
2	11	21	0.524	0.524	0.310	0.333
4	13	23	.565	1.089	.645	.667
6	15	25	.600	1.689	1.000	1.000
8	17	27	.630	2.319	1.373	1.333
10	19	29	.655	2.974	1.761	1.667
12	21	31	.677	3.651	2.162	2.000
14	23	33	.697	4.348	2.574	2.333
16	25	35	.714	5.062	2.997	2.667
18	27	37	.730	5.792	3.429	3.000
20	29	39	.744	6.536	3.870	3.333
22	31	41	.756	7.292	4.317	3.667
24	33	43	.767	8.059	4.771	4.000
26	35	45	.778	8.837	5.232	4.333
28	37	47	.787	9.624	5.698	4.667
30	39	49	.796	10.420	6.169	5.000
32	41	51	.804	11.224	6.645	5.333
34	43	53	.811	12.035	7.125	5.667
36	45	55	.818	12.853	7.610	6.000
38	47	57	.825	13.678	8.098	6.333



Table 13. - Summary of the steps by which the correlation coefficient of 0.734 for the 10th revision was raised to 0.803 for the 12th revision.

<u>Step No</u>	<u>Revision of Munsell spacing</u>	<u>Date</u>	<u>k</u>	<u>Max <math>f_s</math></u>	<u>Correlation coefficient</u>	<u>Standard error of estimate % of mean</u>
25	9th	8/31/67	1.6	1	0.795	13.2
26	9th	"	1.5	1	.783	13.5
-----						
27	10th	11/13/67	1.6	1	.732	14.8
28	10th	"	1.7	1	.734	14.8
29	11th	"	1.6	1	.776	13.7
31	12th	11/14/67	1.5	1	.796	13.2
30	12th	11/15/67	1.6	1	.797	13.1
32	12th	11/14/67	1.7	1	.803	13.0
33	12th	11/15/67	1.8	1	.798	13.1

Table 14. - Munsell re-renotation hue and chroma according to the 12th revision for the 43 colors studied by the OSA Committee on Uniform Color Scales, also the differences from the renotation hue and chroma given in Table 2.

Sample No	Re-renotation		Difference between re-renotation and renotation hue and chroma	
	Hue	Chroma	$\Delta H$	$\Delta C$
1	0.1G	/13.1	-0.3	+0.7
2	1.5G	/9.8	-0.2	-0.2
3	7.7GY	/9.2	-0.5	+0.7
4	4.4GY	/9.7	-0.8	+1.2
5	5.6G	/7.9	0.0	-0.9
6	1.5G	/7.0	-0.2	-0.2
7	6.2GY	/6.7	-0.6	+0.5
8	1.5GY	/7.1	-0.4	+1.3
9	5.9Y	/9.0	+0.2	+1.3
10	3.0BG	/6.6	-1.0	+1.0
11	8.2G	/5.1	-0.2	-0.4
12	0.3G	/4.2	-0.4	+0.2
13	2.2GY	/4.8	-0.2	+1.1
14	5.0Y	/5.9	+0.3	+1.1
15	0.4Y	/8.1	-0.2	+1.1
16	8.5YR	/9.9	-0.1	+0.9
17	9.3BC	/6.6	-2.0	+0.1
18	7.3BG	/4.9	-1.9	+0.0
19	3.3BG	/3.1	-1.1	+0.5
20	5.2GY	/2.0	-1.1	+0.6
21	1.3Y	/3.6	-0.2	+1.1
22	7.4YR	/5.6	-0.4	+0.8
23	5.4YR	/7.8	+0.5	+0.7
24	4.5YR	/10.4	+0.4	+0.8
25	6.7B	/7.4	-3.4	+1.0
26	8.0B	/4.5	-3.3	+0.3
27	3.3PB	/1.4	-2.6	-0.1
28	9.1R	/1.4	+1.6	-0.1
29	1.5YR	/3.9	+1.8	+0.2
30	2.0YR	/5.8	+1.4	+0.1
31	1.6YR	/8.8	+1.6	+0.2
32	1.3YR	/10.6	+1.6	+0.2
33	4.7PB	/8.0	-2.3	+0.6
34	8.3PB	/4.9	-1.6	-0.2
35	7.3P	/3.2	+0.9	-0.9
36	9.2RP	/2.8	+3.9	-1.4
37	5.3R	/4.6	+2.8	-1.0
38	7.3R	/7.1	+2.5	-1.1
39	8.1R	/9.3	+2.5	-1.0
40	6.1P	/6.7	+0.4	-1.4
41	2.2RP	/5.6	+2.2	-2.3
42	7.7RP	/6.2	+3.6	-3.0
43	10.0RP	/7.3	+3.5	-2.9

Table 15, - Predictions of the perceived sizes of 107 differences identified in Table 3 computed by formula (2) from Munsell re-notation hue and chroma of the 43 samples given in Table 14. Differences in Munsell value are neglected.

Diff. No.	Sample Nos.	Hue diff. AH	$f_g h$ from Table 8 for $k=1.7$	$(C_1 C_2)^{1/2}$	Product of columns (4) and (5)	Hue comp. (3)x(6)	Chroma diff. AC	Predicted size of color difference AE
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1-2	1.4	0.107	11.3	1.21	1.69	3.3	3.7
2	1-3	2.4	.106	11.0	1.17	2.81	3.9	4.8
3	2-3	3.8	.105	9.5	1.00	3.80	0.6	3.8
4	2-5	4.1	.105	8.8	.92	3.77	1.9	4.2
5	2-6	0.0	--	8.3	.89	0.00	2.8	2.8
7*	2-12	1.2	.107	6.4	.68	0.82	5.0	5.1
8	3-4	3.3	.106	9.4	1.00	3.30	0.5	3.3
9	3-6	3.8	.105	8.0	.84	3.19	2.2	3.9
10	3-7	1.5	.107	7.8	.83	1.24	2.5	2.8
11	4-7	1.8	.106	8.1	.86	1.55	3.0	3.4
13	4-8	2.9	.106	8.3	.88	2.55	2.6	3.6
15	5-6	4.1	.105	7.4	.78	3.20	0.9	3.3
16	5-10	7.4	.102	7.2	.73	5.40	1.3	5.5
17	5-11	2.6	.106	6.3	.67	1.74	2.8	3.3
18	6-7	5.3	.104	6.8	.71	3.76	0.3	3.8
20	6-11	6.7	.102	6.0	.61	4.09	1.9	4.5
21	6-12	1.2	.107	5.4	.58	0.70	2.8	2.9
22	7-8	4.7	.104	6.9	.72	3.38	0.4	3.4
23	7-12	4.1	.105	5.3	.56	2.30	2.5	3.4
25	7-13	4.0	.105	5.7	.60	2.40	1.9	3.0
26	8-9	5.6	.103	8.0	.82	4.59	1.9	4.9
27	8-13	0.7	.107	5.8	.62	0.43	2.3	2.3
29	8-14	6.5	.103	6.5	.67	4.36	1.2	4.5
31	9-14	0.9	.107	7.3	.78	0.70	3.1	3.2
32	9-15	5.5	.104	8.5	.88	4.84	0.9	4.9
33	10-11	4.8	.104	5.8	.60	2.88	1.5	3.2
34	10-17	6.3	.103	6.6	.68	4.28	0.0	4.3
35	10-18	4.3	.105	5.7	.60	2.58	1.7	3.1
36	11-12	7.9	.101	4.6	.46	3.63	0.9	3.7
38	11-18	9.1	.100	5.0	.50	4.55	0.2	4.6

Table 15. (Continued)

Diff. No.	Sample Nos.	Hue diff. $\Delta H$	$f_s$ from Table 8 for $k=1.7$	$(G_1 C_2)^{1/2}$	Product of columns (4) and (5)	Hue corr. (3)x(6)	Chroma diff. $\Delta C$	Predicted size of color difference $\Delta F$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
39	11-19	5.1	.104	4.0	.42	2.14	2.0	2.9
40	12-13	8.1	.101	4.5	.45	3.64	0.6	3.7
42	12-19	13.0	.092	3.6	.33	4.29	1.1	4.4
44	12-20	5.1	.104	2.9	.30	1.53	2.2	2.7
45	13-14	7.2	.102	5.3	.54	3.89	1.1	4.0
47	13-20	3.0	.106	3.1	.33	0.99	2.3	3.0
49	13-21	10.9	.096	4.2	.40	4.36	1.2	4.5
50	14-15	4.6	.104	6.9	.72	3.31	2.2	4.0
51	14-21	3.7	.105	4.6	.48	1.78	2.3	2.9
52	14-22	7.6	.101	5.7	.58	4.41	0.3	4.4
54	15-16	1.9	.106	9.0	.95	1.80	1.8	2.5
56	15-22	3.0	.106	6.7	.71	2.13	2.5	3.3
57	15-23	5.0	.104	8.0	.83	4.15	0.8	4.2
59	16-23	3.1	.106	8.8	.93	2.88	2.1	3.6
60	16-24	4.0	.105	10.1	1.06	4.24	0.5	4.3
61	17-19	2.0	.106	5.8	.61	1.22	1.7	2.1
62	17-25	7.4	.102	7.0	.71	5.25	0.8	5.3
63	18-19	4.0	.105	3.9	.41	1.64	1.8	2.4
64	18-25	9.4	.100	6.0	.60	5.64	2.5	6.2
66	18-26	10.7	.096	4.7	.45	4.82	0.4	4.8
67	19-20	18.1	.083	2.5	.208	3.76	1.1	3.9
68	19-26	14.7	.088	3.7	.33	4.85	1.4	5.0
70	19-27	20.6	.079	2.1	.166	3.32	1.7	3.7
72	20-21	13.9	.090	2.7	.24	3.34	1.6	3.7
73	20-27	38.1	.051	1.7	.087	3.31	0.6	3.4
74	20-28	26.1	.068	1.7	.116	3.03	0.6	3.1
76	21-22	3.9	.105	4.5	.47	1.83	2.0	2.7
77	21-23	12.2	.094	2.2	.207	2.53	2.2	3.4
79	21-29	9.3	.098	3.7	.36	3.53	0.3	3.5
81	22-23	2.0	.106	6.6	.70	1.40	2.2	2.6
83	22-29	5.9	.103	4.7	.48	2.83	1.7	3.3
84	22-30	5.4	.104	5.7	.59	3.19	0.2	3.2
85	23-24	0.9	.107	9.0	.96	0.86	2.6	2.7
86	23-30	3.4	.106	6.7	.71	2.41	2.0	3.1
87	23-31	3.8	.105	8.3	.87	3.31	1.0	3.4
88	24-31	2.9	.106	9.6	1.02	2.96	1.6	3.4
90	24-32	3.2	.106	10.5	1.11	3.55	0.2	3.6
91	25-26	1.3	.107	5.8	.62	0.81	2.9	3.0
93	25-33	8.0	.101	7.7	.78	6.24	0.6	6.3
95	26-27	5.3	.104	2.5	.26	1.38	3.1	3.4

Table 15. (Continued)

Diff. No.	Sample Nos.	Hue diff. $\Delta H$	$f_g f_h$ from Table 8 for $k=1.7$	$(C_1 C_2)^{1/2}$	Product of columns (4) and (5)	Hue comp. (3)x(6)	Chroma diff. $\Delta C$	Predicted size of color difference $\Delta E$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
96	26-33	6.7	.102	6.0	.61	4.09	3.5	5.4
97	26-34	10.3	.098	4.7	.46	4.74	0.4	4.8
99	27-28	35.8	.054	1.4	.076	2.72	0.0	2.7
100	27-34	5.0	.104	2.6	.27	1.35	3.5	3.8
102	27-35	14.0	.090	2.1	.189	2.65	1.8	3.2
103	28-29	2.4	.106	2.3	.24	0.58	2.5	2.6
104	28-35	21.8	.075	2.1	.158	3.44	1.8	3.9
105	28-36	9.9	.098	2.0	.196	1.94	1.4	2.4
107	29-30	0.5	.107	4.8	.51	0.26	1.9	1.9
109	29-36	12.3	.094	3.3	.31	3.81	1.1	4.0
110	29-37	6.2	.103	4.2	.43	2.67	0.7	2.7
111	30-31	0.4	.107	7.1	.76	0.30	3.0	3.0
113	30-37	6.7	.102	5.2	.53	3.55	1.2	3.7
115	30-38	4.7	.104	6.4	.67	3.15	1.3	3.4
117	31-32	0.3	.107	9.7	1.04	0.31	1.8	1.8
118	31-38	4.3	.105	7.8	.82	3.53	1.7	3.9
119	31-39	3.5	.105	9.0	.94	3.29	0.5	3.3
120	32-39	3.2	.106	9.9	1.05	3.36	1.3	3.6
122	33-34	3.6	.105	6.3	.66	2.38	3.1	3.9
124	34-35	9.1	.100	4.0	.40	3.64	1.7	4.0
125	34-40	7.8	.101	5.7	.58	4.52	1.8	4.8
127	35-36	11.9	.094	3.0	.28	3.33	0.4	3.3
128	35-40	1.2	.107	4.6	.49	0.59	3.5	3.6
129	35-41	4.9	.104	4.2	.44	2.16	2.4	3.2
131	36-37	6.1	.103	3.6	.37	2.26	1.8	2.9
133	36-41	7.0	.102	4.0	.41	2.87	2.8	4.0
134	36-42	1.5	.106	4.2	.45	0.68	3.4	3.5
135	37-38	2.0	.106	5.7	.60	1.20	2.5	2.8
136*	37-39	2.8	.106	6.5	.69	1.93	4.7	5.1
137	37-42	7.6	.101	5.3	.54	4.10	1.6	4.4
138	37-43	5.3	.104	5.8	.60	3.18	2.7	4.1
139	38-39	0.8	.107	8.1	.87	0.70	2.2	2.3
141	38-43	7.3	.102	7.2	.73	5.33	0.2	5.3
143	40-41	6.1	.103	6.1	.63	3.84	1.1	4.0
144	41-42	5.5	.103	5.9	.61	3.36	0.6	3.4
146*	41-43	7.8	.101	6.4	.65	5.07	1.7	5.3
147	42-43	2.3	.106	6.7	.71	1.63	1.1	2.0

Table 16. - Derivation of the factors,  $Y_{V=6}^{1/3} / Y_V^{1/3}$ , by which the colorimetric purities,  $p_c$ , defining the 6/6 chroma locus were multiplied to derive those defining the other nine chroma loci V/6.

<u>V</u>	<u><math>Y_V</math></u>	<u><math>Y_V^{1/3}</math></u>	<u><math>Y_{V=6}^{1/3} / Y_V^{1/3}</math></u>
10.0	102.57	4.681	0.664
9.0	78.66	4.285	.726
8.0	59.10	3.895	.798
7.0	43.06	3.505	.887
6.0	30.05	3.109	1.000
5.0	19.77	2.704	1.150
4.0	12.00	2.289	1.358
3.0	6.555	1.872	1.661
2.0	3.126	1.462	2.127
1.0	1.210	1.066	2.917

Table 17. - Tentative re-renotations for the 7.5PB 3/ renotation  
 chroma scale; also re-renotations.

<u>Renotation</u>	<u>Chromaticity Coordinates</u>		<u>Tentative</u>	<u>Re-renotation</u>
	<u>x</u>	<u>y</u>	<u>re-renotation</u>	<u>chroma</u>
7.5PB 3/2	0.2777	0.2687	/2.8	/2.8
3/4	.2520	.2319	/5.6	/5.6
3/6	.2311	.2010	/8.2	/8.2
3/8	.2149	.1761	/11.0	/10.8
3/10	.2005	.1536	/14.2	/13.3
3/12	.1903	.1353	/17.3	/15.1
3/14	.1824	.1188	/20.7	/16.6
3/16	.1765	.1048	/24.4	/18.0
3/18	.1730	.0948	/27.9	/19.1
3/20	.1702	.0867	/31.0	/19.9
3/22	.1677	.0782	/34.3	/20.7
3/24	.1658	.0711	/38.3	/21.6
3/26	.1642	.0655	/42.0	/22.1









V/C	2.5YR			5.0YR			7.5YR			10.0YR		
	Y	x	y	Y	x	y	Y	x	y	Y	x	y
10/16												
14										87.7	0.444 <sub>5</sub>	0.430
12							88.4	0.431*	0.403*	89.2	.424	.414
10				89.2	0.415*	0.377*	90.8	.410	.389 <sub>5</sub>	91.6	.405	.399
8	90.0	0.396 <sub>5</sub> *	0.352 <sub>5</sub> *	91.6	.394	.365 <sub>5</sub>	93.2	.390	.376 <sub>5</sub>	94.1	.386	.382 <sub>5</sub>
6	93.2	.374	.343	95.0	.372	.353	95.0	.369	.362	95.9	.366 <sub>5</sub>	.366 <sub>5</sub>
4	96.7	.349 <sub>5</sub>	.333	97.6	.348	.340	98.7	.345 <sub>5</sub>	.344 <sub>5</sub>	98.7	.344	.348
2	100.5	.328	.324	100.5	.326 <sub>5</sub>	.327 <sub>5</sub>	100.5	.326	.329	100.5	.324 <sub>5</sub>	.331
0	102.5 <sub>7</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>									
9/14												
12										67.3	.438	.424
10							68.4	.422 <sub>5</sub>	.398	69.6	.417	.407 <sub>5</sub>
8	68.4	.406*	.357*	69.6	.404	.370 <sub>5</sub>	70.2	.400	.382 <sub>5</sub>	70.9	.395	.391 <sub>5</sub>
6	70.9	.381	.347	72.1	.379 <sub>5</sub>	.358	72.8	.376 <sub>5</sub>	.366 <sub>5</sub>	72.8	.373	.373 <sub>5</sub>
4	73.5	.353 <sub>5</sub>	.335	74.2	.352	.342 <sub>5</sub>	74.9	.349 <sub>5</sub>	.347 <sub>5</sub>	75.7	.347	.352
2	76.4	.330	.325 <sub>5</sub>	77.1	.329	.329	77.1	.328	.331	77.1	.326 <sub>5</sub>	.333 <sub>5</sub>
0	78.6 <sub>6</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>									
8/18				43.8	.546*	.431*	45.1	.541*	.463 <sub>5</sub> *			
16				45.1	.519 <sub>5</sub>	.422	46.5	.514	.449	47.3	.506	.469 <sub>5</sub>
14	44.8	.493 <sub>5</sub> *	.382 <sub>5</sub> *	46.5	.493 <sub>5</sub>	.411	47.6	.489	.435	48.5	.482	.452 <sub>5</sub>
12	46.5	.467 <sub>5</sub>	.375	48.0	.467	.399	49.2	.462 <sub>5</sub>	.419	50.1	.456	.434
10	48.5	.440	.365	49.6	.440	.386	50.5	.436 <sub>5</sub>	.403	50.9	.431 <sub>5</sub>	.415 <sub>5</sub>
8	50.5	.416	.358	51.4	.415	.373	52.3	.412	.387	52.8	.407	.398
6	52.3	.389	.348 <sub>5</sub>	53.2	.388	.360	54.2	.385 <sub>5</sub>	.369 <sub>5</sub>	54.2	.382	.378
4	54.7	.358 <sub>5</sub>	.336 <sub>5</sub>	55.3	.358	.343 <sub>5</sub>	55.7	.356	.349	55.7	.353	.353 <sub>5</sub>
2	57.4	.333	.326	57.4	.332	.329	57.4	.331	.331 <sub>5</sub>	57.9	.329 <sub>5</sub>	.333
0	59.1 <sub>0</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>									
7/20												
18	29.1	.578 <sub>5</sub>	.398 <sub>5</sub>	30.3	.578*	.436*						
16	30.1	.549	.392	31.4	.547	.427	32.6	.542	.455	33.4	.535*	.482*
14	31.4	.519	.385	32.6	.517 <sub>5</sub>	.416	33.6	.513 <sub>5</sub>	.441 <sub>5</sub>	34.4	.506 <sub>5</sub>	.465 <sub>5</sub>
12	32.6	.488 <sub>5</sub>	.377 <sub>5</sub>	33.9	.488	.404 <sub>5</sub>	35.0	.484	.426	35.6	.477	.445 <sub>5</sub>
10	34.2	.461	.369 <sub>5</sub>	35.3	.460 <sub>5</sub>	.392 <sub>5</sub>	36.2	.456	.411	36.5	.449 <sub>5</sub>	.426 <sub>5</sub>



































V/C	2.5RP			5.0RP			7.5RP			10.0RP		
	Y	x	y	Y	x	y	Y	x	y	Y	x	y
6/20												
18	17.5	0.354 <sub>5</sub>	0.157 <sub>5</sub>	17.4	0.393*	0.172*	17.4	0.442*	0.196*			
16	18.3	.350 <sub>5</sub>	.174	18.2	.385 <sub>5</sub>	.187	18.2	.430	.208	18.1	0.478*	0.235 <sub>5</sub> *
14	19.0	.347	.189	18.9	.379	.199 <sub>5</sub>	19.0	.418	.220	19.0	.459 <sub>5</sub>	.246
12	20.0	.342	.206	20.0	.370	.217	20.0	.404	.233	20.0	.440 <sub>5</sub>	.255 <sub>5</sub>
10	21.2	.338	.222 <sub>5</sub>	21.2	.361 <sub>5</sub>	.231 <sub>5</sub>	21.0	.390 <sub>5</sub>	.246	21.0	.420 <sub>5</sub>	.265 <sub>5</sub>
8	22.3	.334	.238	22.3	.353 <sub>5</sub>	.245 <sub>5</sub>	22.3	.377 <sub>5</sub>	.258 <sub>5</sub>	22.3	.400	.276
6	23.6	.329 <sub>5</sub>	.255	23.6	.344	.262	23.6	.362	.272 <sub>5</sub>	23.9	.379	.286
4	25.2	.324	.274	25.2	.334	.279	25.5	.345 <sub>5</sub>	.286 <sub>5</sub>	25.7	.356	.296 <sub>5</sub>
2	27.3	.317 <sub>5</sub>	.294 <sub>5</sub>	27.6	.322 <sub>5</sub>	.297 <sub>5</sub>	27.6	.328	.301 <sub>5</sub>	27.8	.333	.307
0	30.0 <sub>5</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>									
5/20												
18	10.6	.358*	.126*	10.6	.401*	.142*						
16	11.1	.354	.142 <sub>5</sub>	11.1	.393 <sub>5</sub>	.157	11.1	.448*	.084*	11.1	.506 <sub>5</sub> *	.216 <sub>5</sub> *
14	11.6	.350 <sub>5</sub>	.159 <sub>5</sub>	11.6	.386 <sub>5</sub>	.173	11.6	.436	.197	11.5	.490	.225 <sub>5</sub>
12	12.1	.347	.174 <sub>5</sub>	12.1	.380 <sub>5</sub>	.186	12.1	.424	.208 <sub>5</sub>	12.1	.471	.236
10	12.8	.342	.194 <sub>5</sub>	12.8	.371 <sub>5</sub>	.205	12.8	.409 <sub>5</sub>	.223 <sub>5</sub>	12.8	.452	.246
8	13.5	.338	.213	13.4	.363 <sub>5</sub>	.221 <sub>5</sub>	13.4	.396 <sub>5</sub>	.236 <sub>5</sub>	13.4	.430 <sub>5</sub>	.257 <sub>5</sub>
6	14.2	.333 <sub>5</sub>	.228 <sub>5</sub>	14.2	.356	.237	14.2	.382	.250	14.2	.408 <sub>5</sub>	.268 <sub>5</sub>
4	15.2	.329	.247 <sub>5</sub>	15.2	.346 <sub>5</sub>	.255	15.2	.365 <sub>5</sub>	.265 <sub>5</sub>	15.3	.384 <sub>5</sub>	.280 <sub>5</sub>
2	16.3	.323 <sub>5</sub>	.269	16.5	.336	.274	16.5	.348	.282	16.6	.360	.293
0	18.1	.317 <sub>5</sub>	.292	18.0	.323	.295	18.0	.330	.299	18.0	.335 <sub>7</sub>	.304 <sub>5</sub>
4/20												
18	6.2	.356*	.110*		-	-						
16	6.5	.353 <sub>5</sub>	.127	6.4	.400	.138						
14	6.8	.350 <sub>5</sub>	.144	6.7	.392	.155	6.6	.452	.174	6.6	.511	.206 <sub>5</sub>
12	7.1	.346 <sub>5</sub>	.163	7.1	.384	.172 <sub>5</sub>	6.9	.436	.191	6.9	.488 <sub>5</sub>	.220
10	7.5	.343	.180	7.4	.377	.189	7.4	.421	.207	7.3	.466 <sub>5</sub>	.232
8	7.9	.338 <sub>5</sub>	.199 <sub>5</sub>	7.8	.367 <sub>5</sub>	.208 <sub>5</sub>	7.8	.404 <sub>5</sub>	.223 <sub>5</sub>	7.8	.444 <sub>5</sub>	.245

















V/C	2.5B			5.0B			7.5B			10.0B		
	Y	x	y	Y	x	y	Y	x	y	Y	x	y
6/20												
18	18.2	.129 <sub>5</sub>	.203	18.2	.133 <sub>5</sub>	.189	18.2	.156	.179	18.2	.165	.174
16	18.9	.145	.216	18.9	.158 <sub>5</sub>	.203	18.9	.170	.194	18.9	.178 <sub>5</sub>	.188
14	19.7	.162	.229 <sub>5</sub>	19.8	.175	.217 <sub>5</sub>	19.9	.185 <sub>5</sub>	.209 <sub>5</sub>	19.9	.194	.204
12	20.6	.180 <sub>5</sub>	.243 <sub>5</sub>	20.7	.191	.232	20.7	.200	.214 <sub>5</sub>	20.7	.208	.219
10	21.6	.198 <sub>5</sub>	.256 <sub>5</sub>	21.8	.208	.247	21.8	.216	.239 <sub>5</sub>	21.8	.223	.234
8	22.8	.217 <sub>5</sub>	.269	22.8	.224 <sub>5</sub>	.250	22.8	.231	.254	22.8	.237 <sub>5</sub>	.248 <sub>5</sub>
6	24.0	.238 <sub>5</sub>	.238	24.0	.243	.275	24.0	.248	.269	24.0	.253	.264 <sub>5</sub>
4	25.7	.262	.295	25.7	.264 <sub>5</sub>	.288 <sub>5</sub>	25.7	.267 <sub>5</sub>	.284 <sub>5</sub>	25.7	.274	.278
2	27.8	.286 <sub>5</sub>	.306 <sub>5</sub>	27.8	.287 <sub>5</sub>	.303	27.8	.288 <sub>5</sub>	.301	27.6	.290	.299
0	30.0 <sub>5</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>									
5/22							10.6	.112 <sub>5</sub> *	.127 <sub>5</sub> *	10.6	.122 <sub>5</sub> *	.122 <sub>5</sub> *
20	11.0	.094*	.167*	11.1	.111 <sub>5</sub> *	.154*	11.1	.128	.144	11.1	.138	.139 <sub>5</sub>
18	11.4	.112 <sub>5</sub>	.183 <sub>5</sub>	11.5	.128 <sub>5</sub>	.171	11.5	.144	.166 <sub>5</sub>	11.5	.153 <sub>5</sub>	.157
16	11.9	.131	.199	12.0	.145 <sub>5</sub>	.187	12.0	.159 <sub>5</sub>	.178	12.0	.168	.173
14	12.5	.150	.215	12.5	.163	.203 <sub>5</sub>	12.5	.175	.195	12.6	.184	.184 <sub>5</sub>
12	13.1	.168	.229	13.2	.179 <sub>5</sub>	.219 <sub>5</sub>	13.3	.190 <sub>5</sub>	.211 <sub>5</sub>	13.3	.199 <sub>5</sub>	.206
10	13.8	.188 <sub>5</sub>	.245 <sub>5</sub>	13.9	.198	.236 <sub>5</sub>	13.9	.207 <sub>5</sub>	.229	13.9	.215	.224
8	14.5	.208 <sub>5</sub>	.260	14.6	.216	.251 <sub>5</sub>	14.6	.223 <sub>5</sub>	.244 <sub>5</sub>	14.6	.230	.239 <sub>5</sub>
6	15.4	.231	.275	15.4	.236 <sub>5</sub>	.267 <sub>5</sub>	15.4	.241 <sub>5</sub>	.261 <sub>5</sub>	15.4	.246 <sub>5</sub>	.256 <sub>5</sub>
4	16.6	.255 <sub>5</sub>	.289 <sub>5</sub>	16.6	.258 <sub>5</sub>	.284	16.6	.262 <sub>5</sub>	.279 <sub>5</sub>	16.6	.266	.276
2	18.1	.282 <sub>5</sub>	.304	18.0	.284	.301	18.0	.285 <sub>5</sub>	.298 <sub>5</sub>	18.0	.287 <sub>5</sub>	.296
0	19.7 <sub>7</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>									
4/22										6.2	.109*	.100*
20				6.3	.092*	.131 <sub>5</sub> *	6.4	.111 <sub>5</sub> *	.123 <sub>5</sub> *	6.4	.124	.119
18	6.6	.091*	.161*	6.7	.110 <sub>5</sub>	.151	6.7	.128 <sub>5</sub>	.143 <sub>5</sub>	6.7	.141	.139
16	6.9	.111 <sub>5</sub>	.178 <sub>5</sub>	6.9	.128 <sub>5</sub>	.169	7.0	.145	.161 <sub>5</sub>	7.0	.156	.156 <sub>5</sub>
14	7.2	.132 <sub>5</sub>	.197	7.3	.147 <sub>5</sub>	.187 <sub>5</sub>	7.4	.162	.180	7.4	.172 <sub>5</sub>	.176
12	7.6	.154	.215	7.7	.167 <sub>5</sub>	.206 <sub>5</sub>	7.7	.180	.199	7.7	.189	.194

V/C	2.5B			5.0B			7.5B			10.0B		
	Y	x	y	Y	x	y	Y	x	y	Y	x	y
1/16												
14	-	-		0.63	0.088*	0.118*	0.63	0.104	0.112 <sub>5</sub>	0.61	0.106	0.087
12	0.66	0.089	0.144	0.65	.105 <sub>5</sub>	.138 <sub>5</sub>	0.66	.121	.133 <sub>5</sub>	0.63	.123	.107 <sub>5</sub>
10	0.68	.111	.167	0.69	.126	.161	0.70	.140 <sub>5</sub>	.156	0.67	.139 <sub>5</sub>	.128 <sub>5</sub>
8	0.72	.133 <sub>5</sub>	.189	0.73	.146 <sub>5</sub>	.182 <sub>5</sub>	0.73	.158 <sub>5</sub>	.176 <sub>5</sub>	0.70	.157 <sub>5</sub>	.150
6	0.78	.161 <sub>5</sub>	.214 <sub>5</sub>	0.79	.172	.208	0.79	.182 <sub>5</sub>	.202	0.74	.175	.170
4	0.86	.197 <sub>5</sub>	.246	0.86	.205 <sub>5</sub>	.239	0.86	.213	.233	0.79	.195 <sub>5</sub>	.195 <sub>5</sub>
2	0.99	.247	.281	0.99	.251	.276	0.98	.255	.271	0.86	.223	.267
0	1.21 <sub>0</sub>	.310 <sub>1</sub>	.316 <sub>3</sub>							0.98	.259	.267

















